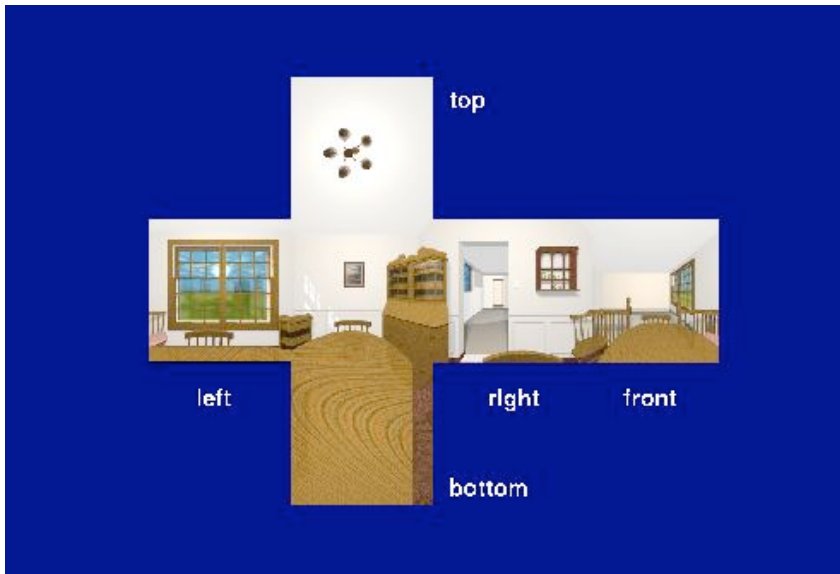


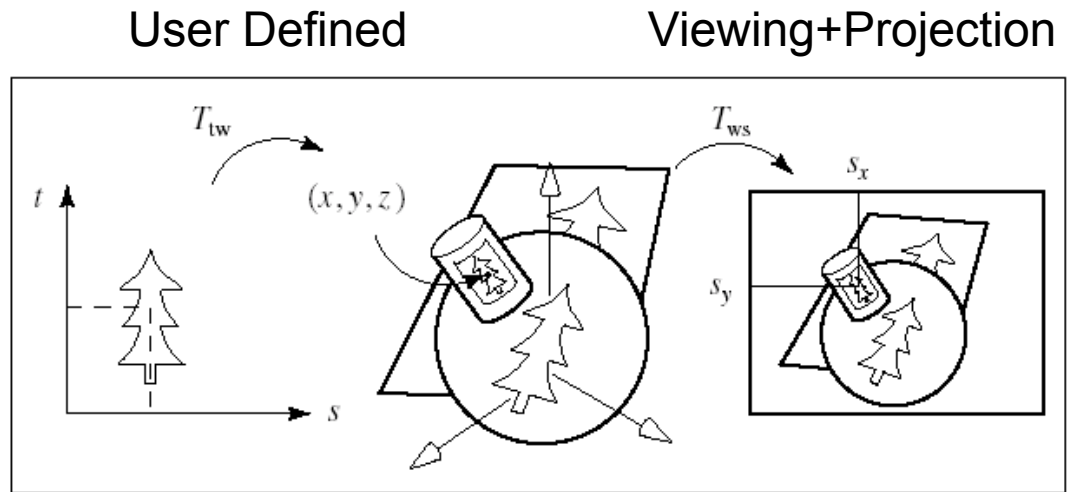
Texture Mapping

Pasting textures on surfaces: Hill 8.5



Systems involved

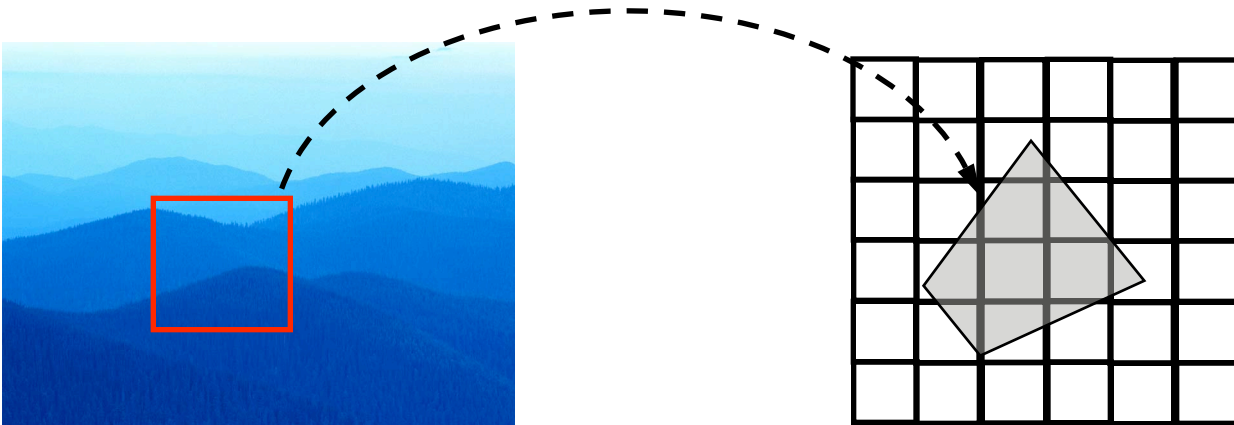
FIGURE 8.35 Drawing texture on several objects of different shape.



$$(s_x, s_y) = T_{ws}(T_{tw}(s, t))$$



Texture to Screen

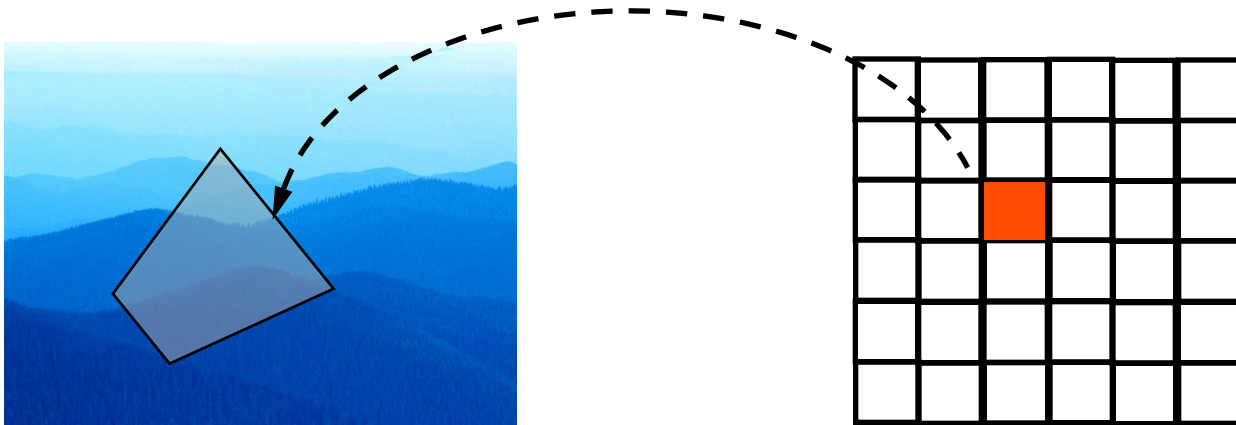


$$(sx, sy) = Tws(Ttw(s, t))$$

We would have to calculate pixel coverages

Screen to texture

Better approach

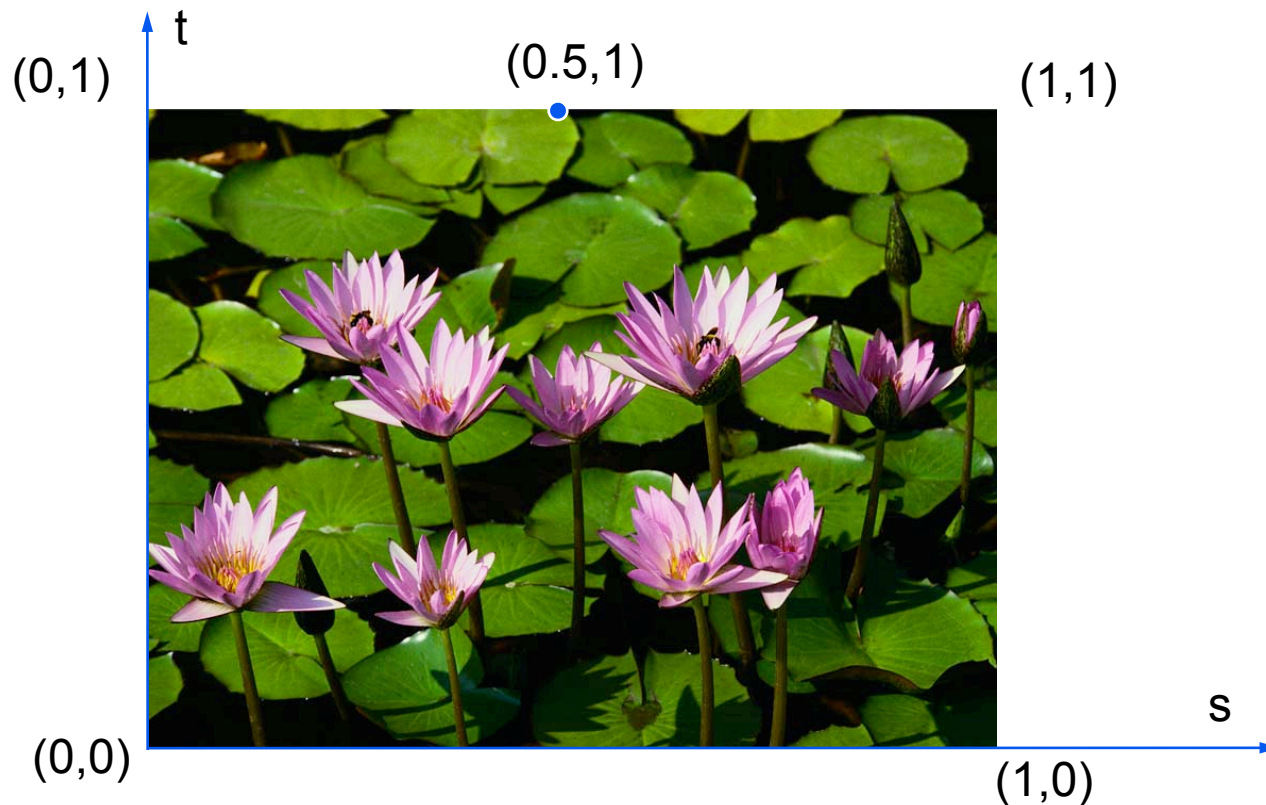


$$(s,t) = Twt(Tsw(sx,sy))$$

Requires inverting the
projection matrix

2D Textures are images

They are always assigned the shown parametric coordinates (s,t).



From texture to world (object)

To apply a texture to an object we have to find a correspondance between (s,t) and some object coordinate system.

- Mapping via a parametric representation of the object space (points).
- By hand.

Mapping from texture to a parametric representation of the object space

Linear transformation

Texture space (s,t) to object space (u,v)

$$u = u(s,t) = a_u s + b_u t + c_u$$

$$v = v(s,t) = a_v s + b_v t + c_v$$

$$s \text{ in } [0,1]$$

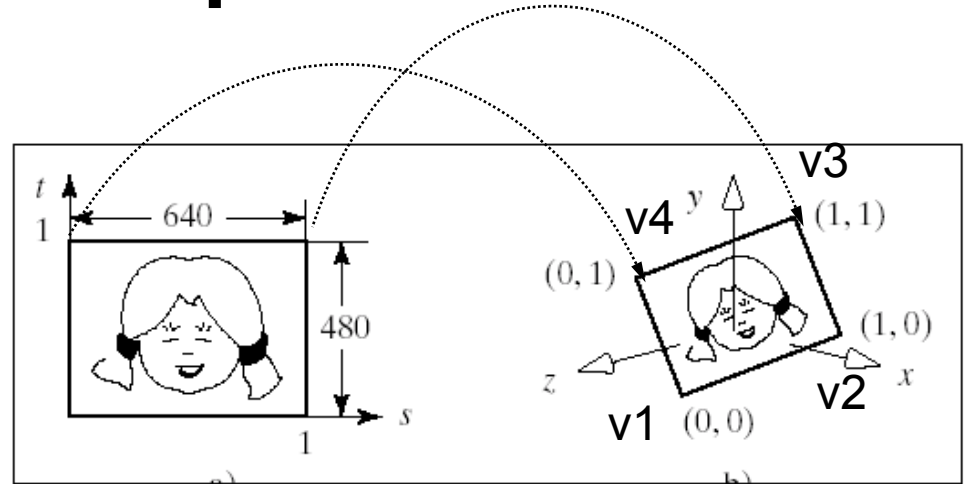
$$t \text{ in } [0,1]$$

Example: Image to a quadrilateral

Simply

$$u = u(s,t) = s$$

$$v = v(s,t) = t$$



```
glTexCoord2f(0,0) ; glVertex3dv(v1) ;
```

```
glTexCoord2f(1,0) ; glVertex3dv(v2) ;
```

```
glTexCoord2f(1,1) ; glVertex3dv(v3) ;
```

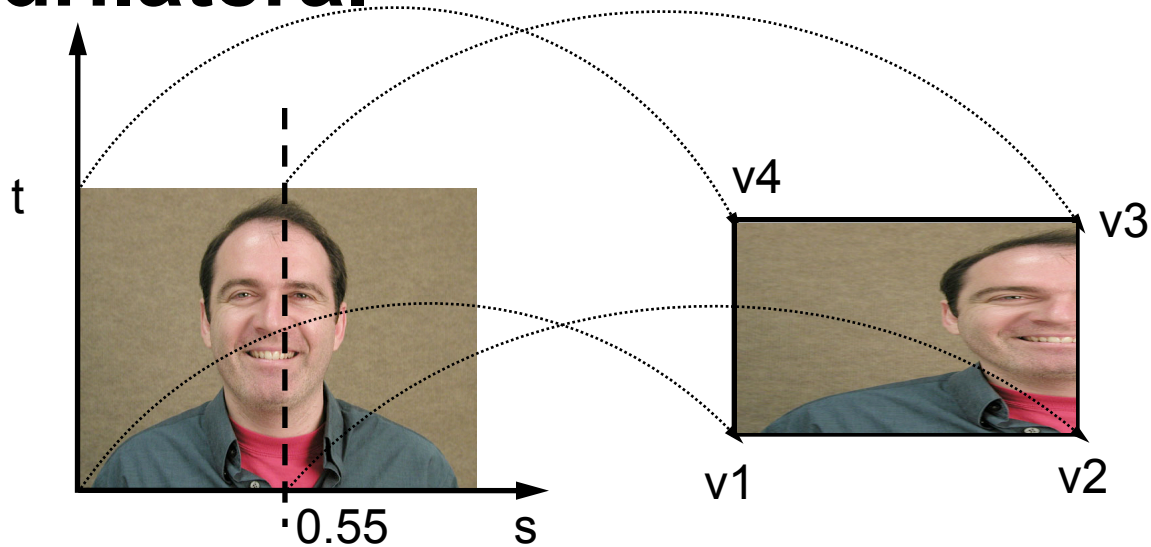
```
glTexCoord2f(0,1) ; glVertex3dv(v4) ;
```


Example 2: Piece of image to a quadrilateral

Use only left part

$$u = u(s,t) = 0.55s$$

$$v = v(s,t) = t$$



```
glTexCoord2f(0,0) ; glVertex3dv(v1) ;
```

```
glTexCoord2f(0.55,0) ; glVertex3dv(v2) ;
```

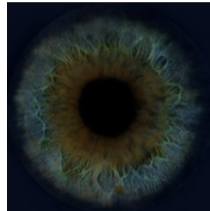
```
glTexCoord2f(0.55,1) ; glVertex3dv(v3) ;
```

```
glTexCoord2f(0,1) ; glVertex3dv(v4) ;
```

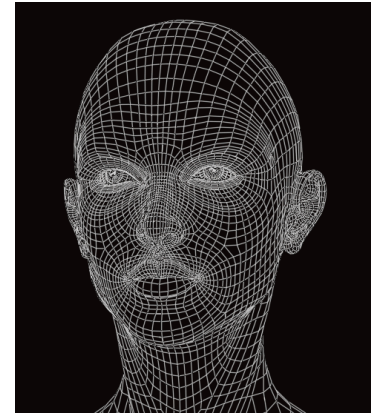
Packing textures for efficiency

Example 3: Many texture maps, thousands of vertices and texture coordinates

Texture maps



+

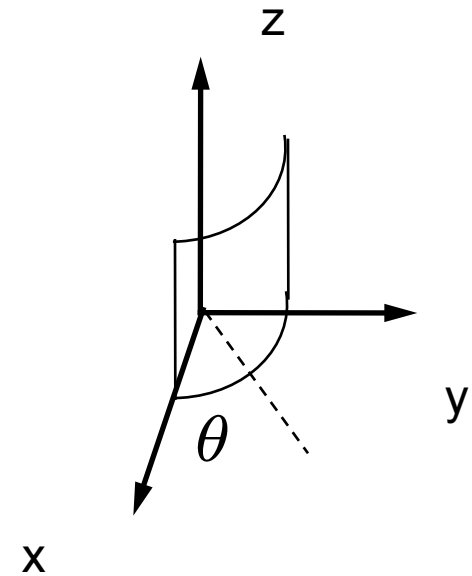
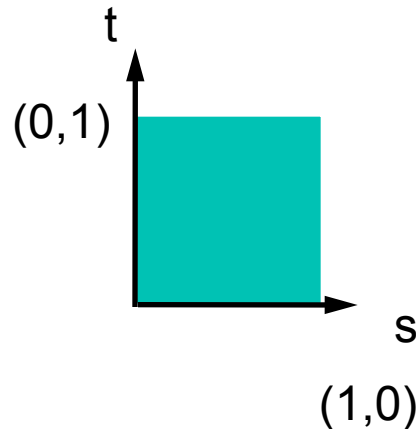


+ lighting =



Example: Square texture to cylinder

Cylinder has height 1



Parametric form :

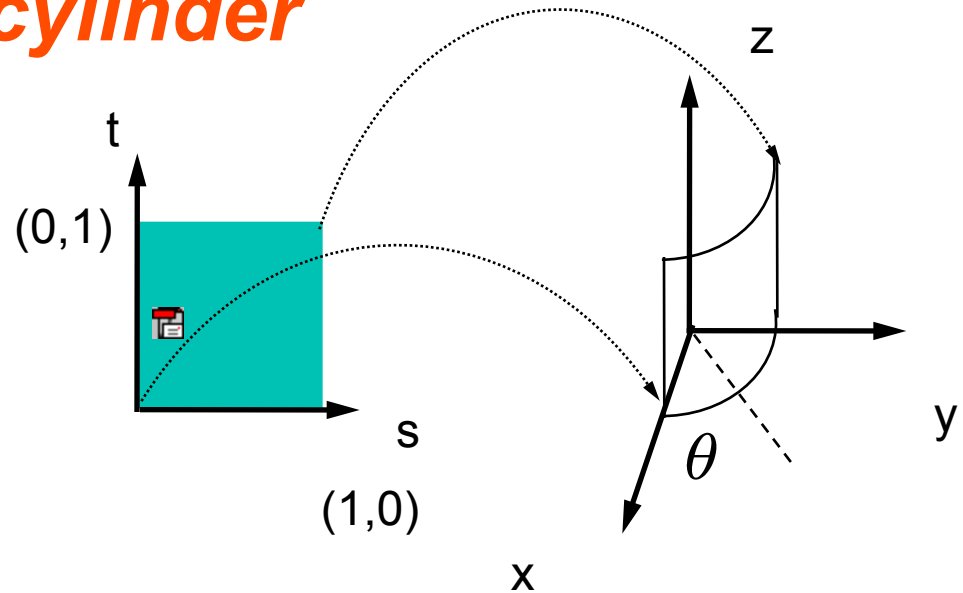
$$x = r \cos \theta, \quad y = r \sin \theta, \quad z$$

Surface parameters : $u = \theta, v = z$

with $0 \leq u \leq \pi/2, 0 \leq v \leq 1$

Example : Square texture to cylinder

Square texture to cylinder

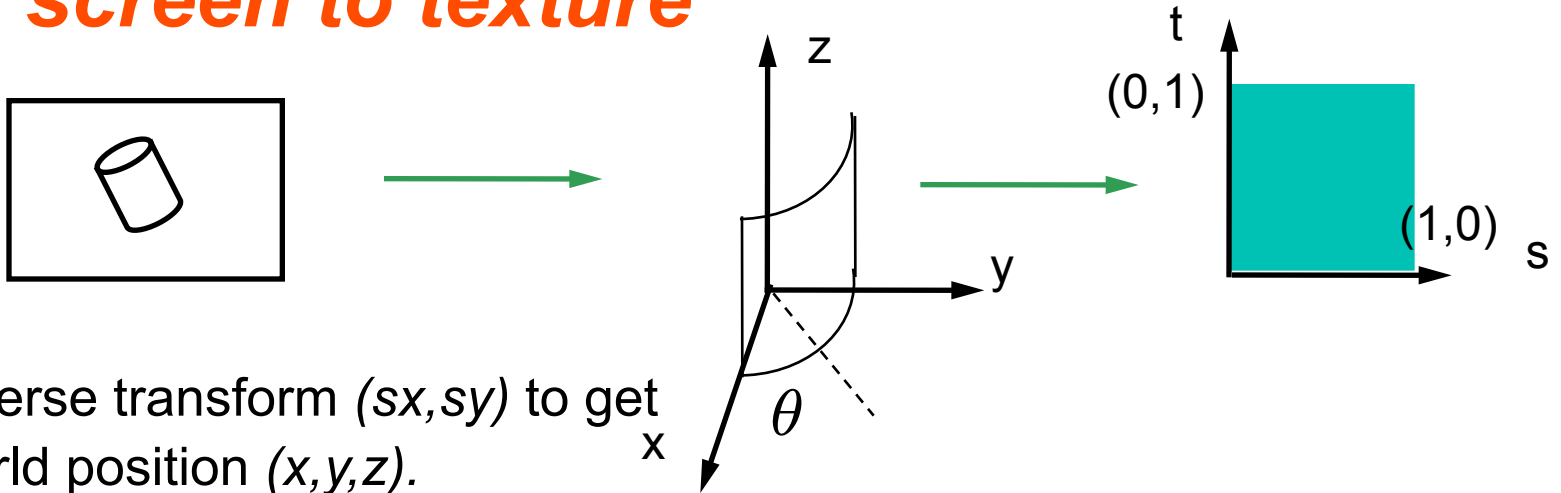


We pick the following linear transformation that maps $(s, t) = (0, 0)$ to $(x, y, z) = (r, 0, 0)$ and $(s, t) = (1, 1)$ to $(x, y, z) = (0, r, r)$.

$$u = s\pi/2, \quad v = t$$

Example : Square texture to cylinder

From screen to texture



1. Inverse transform (sx, sy) to get world position (x, y, z) .
2. Then having (x, y, z)

$$u = \tan^{-1}(y/x), \quad v = z$$

$$s = 2u/\pi, \quad t = v$$

$$\text{Reminder : } u = s\pi/2, \quad v = t$$

$$x = r\cos\theta, \quad y = r\sin\theta, \quad z$$

$$\text{Surface parameters : } u = \theta, v = z$$

How does that work with the graphics pipeline?

Only polygons

Only vertices go down the graphics pipeline.

Interior points?

Calculate texture coordinates by interpolation along scanlines.

Rendering the texture

Scanline in screen space

- Generating s, t coordinates for each pixel

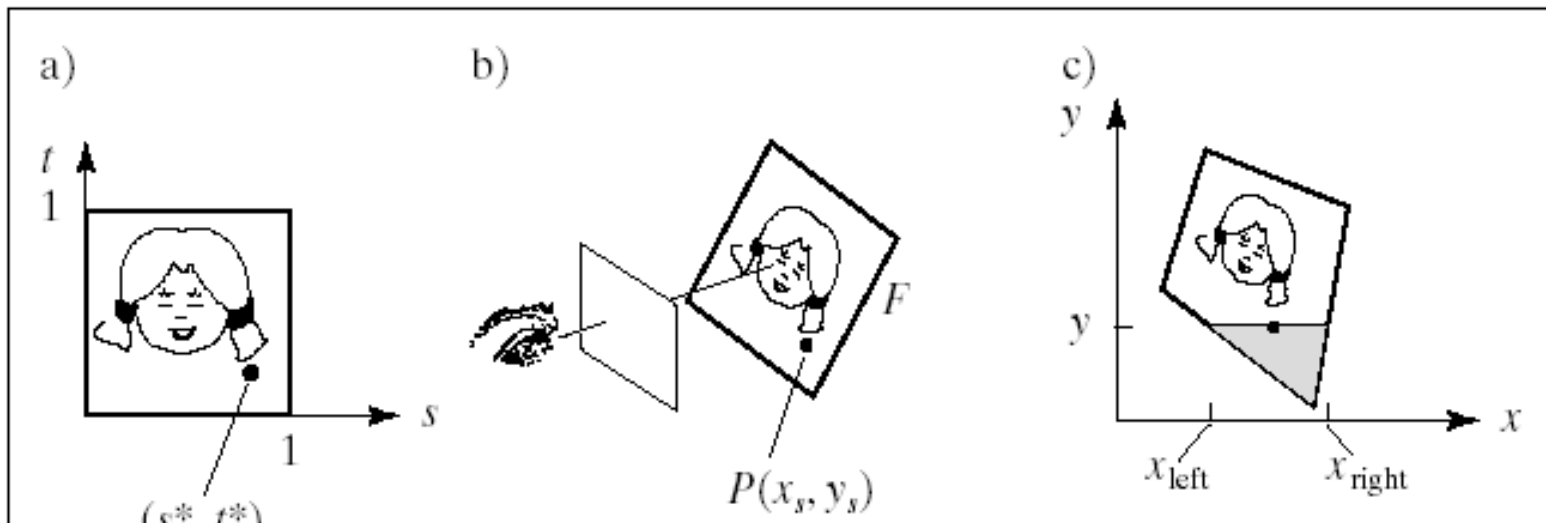


FIGURE 8.39 Rendering a face in a camera snapshot.

Interpolation of texture coordinates

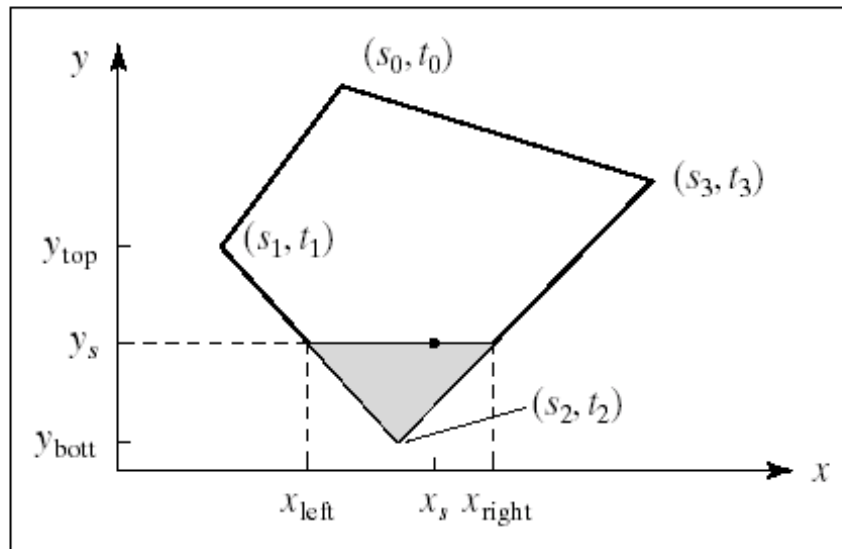


FIGURE 8.40 Incremental calculation of texture coordinates.

Problem

Perspective forshortening

- Scanconversion takes equal steps along scanline (screen space)
- Equal steps in screen space not equal steps in world space

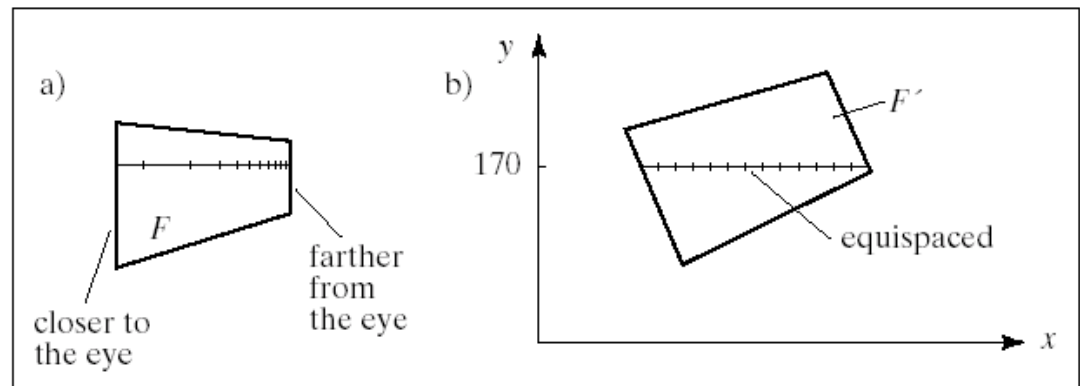
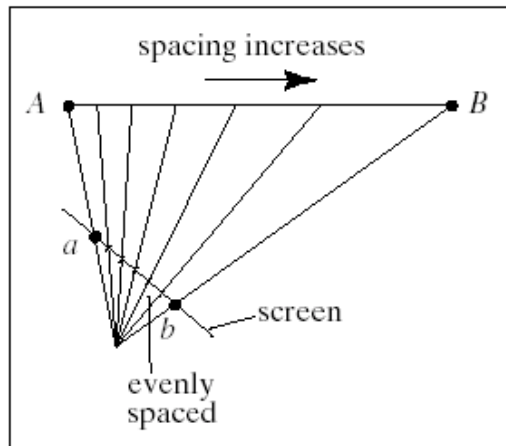
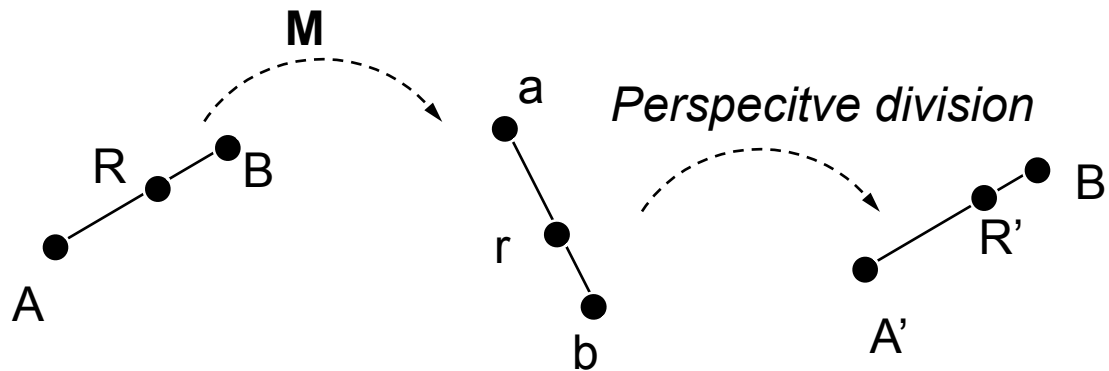


FIGURE 8.41 Spacing of samples with linear interpolation.

Inbetween points

How do points on lines transform?



$$R(g) = (1-g)A + gB$$

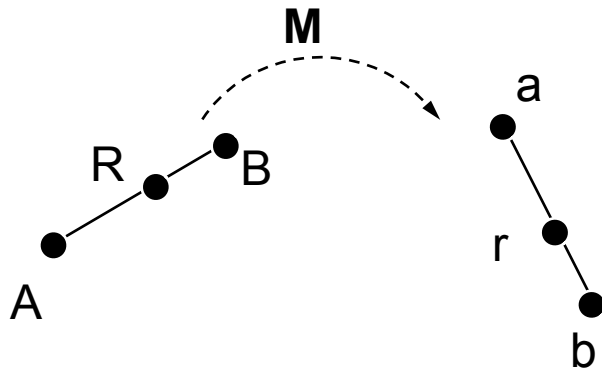
$$r = MR$$

$$R'(f) = (1-f)a' + fb'$$
 in cartesian coordinates

What is the relationship between g and f ?

First step

World to homogeneous space (4D)



$$R = (1 - g)A + gB$$

$$r = MR = M[(1 - g)A + gB] = (1 - g)MA + gMB \Rightarrow$$

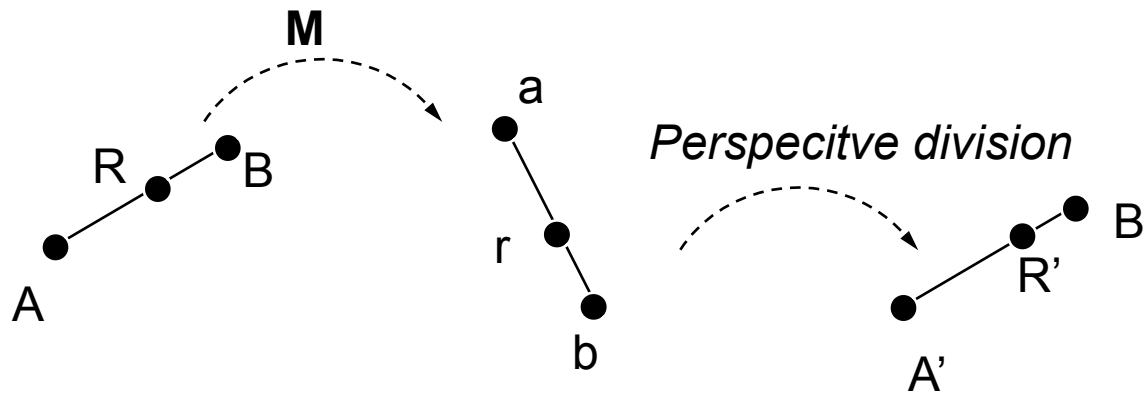
$$r = (1 - g)a + gb$$

$$a = MA = (a_1, a_2, a_3, a_4)$$

$$b = MB = (b_1, b_2, b_3, b_4)$$

Second step

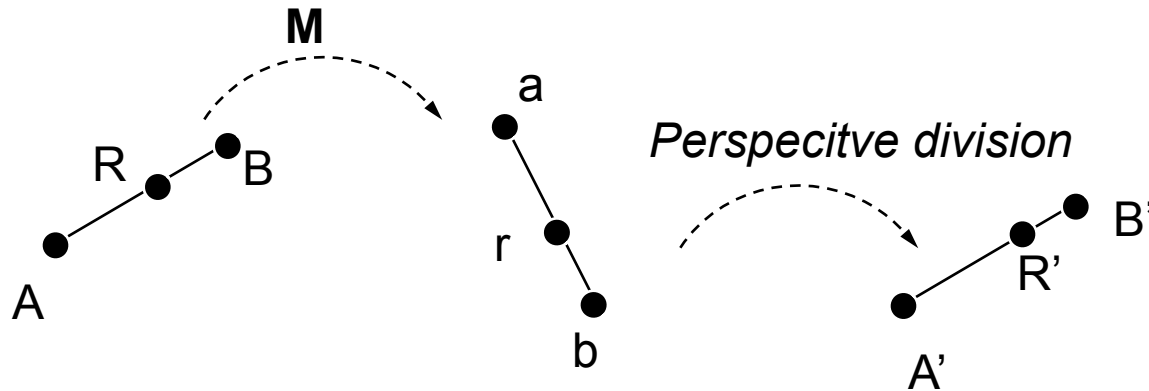
Perspective division



$$\left\{ \begin{array}{l} r = (1 - g)a + gb \\ a = (a_1, a_2, a_3, a_4) \\ b = (b_1, b_2, b_3, b_4) \end{array} \right\} \Rightarrow$$

$$R'_1 = \frac{r_1}{r_4} = \frac{(1 - g)a_1 + gb_1}{(1 - g)a_4 + gb_4}$$

Putting all together



$$R'_1 = \frac{(1-g)a_1 + gb_1}{(1-g)a_4 + gb_4} = \frac{\text{lerp}(a_1, b_1, g)}{\text{lerp}(a_4, b_4, g)}$$

At the same time :

$$R' = (1-f)A' + fB' \Rightarrow$$

$$R'_1 = (1-f)\frac{a_1}{a_4} + f\frac{b_1}{b_4} = \text{lerp}\left(\frac{a_1}{a_4}, \frac{b_1}{b_4}, f\right)$$



Relation between the fractions

$$\left. \begin{aligned} R_1(f) &= \frac{\text{lerp}(a_1, b_1, g)}{\text{lerp}(a_4, b_4, g)} \\ R_1(f) &= \text{lerp}\left(\frac{a_1}{a_4}, \frac{b_1}{b_4}, f\right) \end{aligned} \right\} \Rightarrow g = \frac{f}{\text{lerp}\left(\frac{b_4}{a_4}, 1, f\right)}$$

substituting this in $R(g) = (1 - g)A + gB$ yields

$$R_1 = \frac{\text{lerp}\left(\frac{A_1}{a_4}, \frac{B_1}{b_4}, f\right)}{\text{lerp}\left(\frac{1}{a_4}, \frac{1}{b_4}, f\right)}$$

THAT MEANS: For a given f in **screen space** and A, B in **world space** we can find the corresponding R (or g) in **world space** using the above formula.

“ A ” can be texture coordinates, position, color, normal etc.

Rendering images incrementally

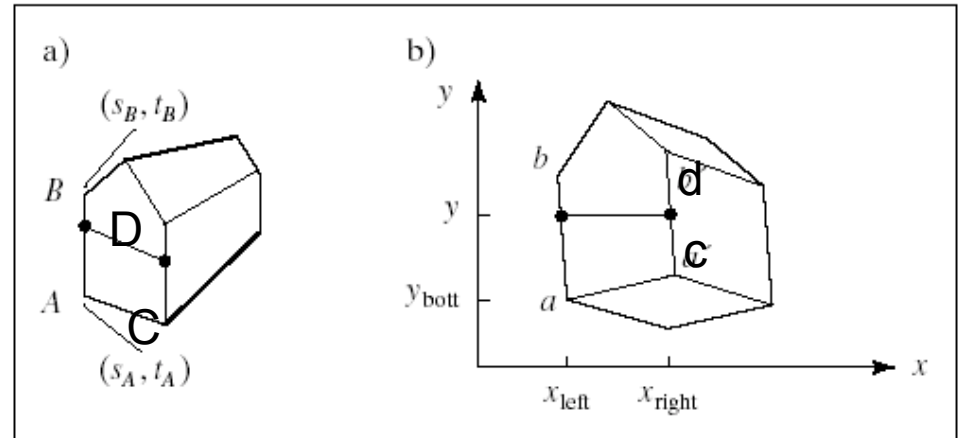
A maps to a (homogeneous)

B maps to b

C maps to c

D maps to d

For scanline y and two edges:



$f_{edge} = (y - y_{bott}) / (y_{top} - y_{bott})$ so for the left and right edges :

$$s_{left}(y) = \frac{\text{lerp}\left(\frac{s_A}{a_4}, \frac{s_B}{b_4}, f_l\right)}{\text{lerp}\left(\frac{1}{a_4}, \frac{1}{b_4}, f_l\right)}, s_{right}(y) = \frac{\text{lerp}\left(\frac{s_C}{c_4}, \frac{s_D}{d_4}, f_r\right)}{\text{lerp}\left(\frac{1}{c_4}, \frac{1}{d_4}, f_r\right)}$$

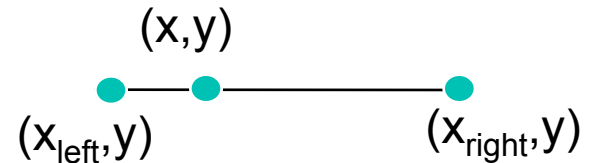
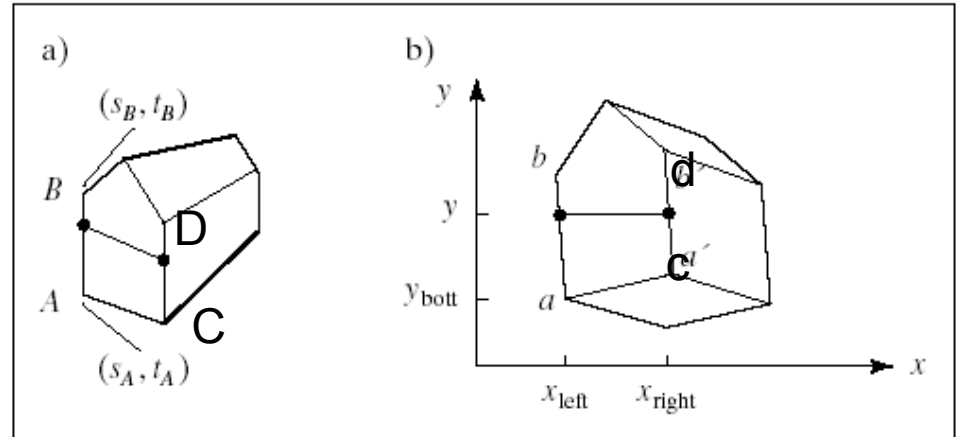
Once we have s_{left} and s_{right} another hyperbolic interpolation fills in the scanline

Interpolation along the scanline

$$s_{left}(y) = \frac{\text{lerp}\left(\frac{s_A}{a_4}, \frac{s_B}{b_4}, f_l\right)}{\text{lerp}\left(\frac{1}{a_4}, \frac{1}{b_4}, f_l\right)},$$

$$s_{right}(y) = \frac{\text{lerp}\left(\frac{s_C}{c_4}, \frac{s_D}{d_4}, f_r\right)}{\text{lerp}\left(\frac{1}{c_4}, \frac{1}{d_4}, f_r\right)}$$

$$s(x, y) = \frac{\text{lerp}\left(\frac{s_{left}}{h_{left}}, \frac{s_{right}}{h_{right}}, f\right)}{\text{lerp}\left(\frac{1}{h_{left}}, \frac{1}{h_{right}}, f\right)}$$



What are the f , and h 's?

Interpolation along the scanline

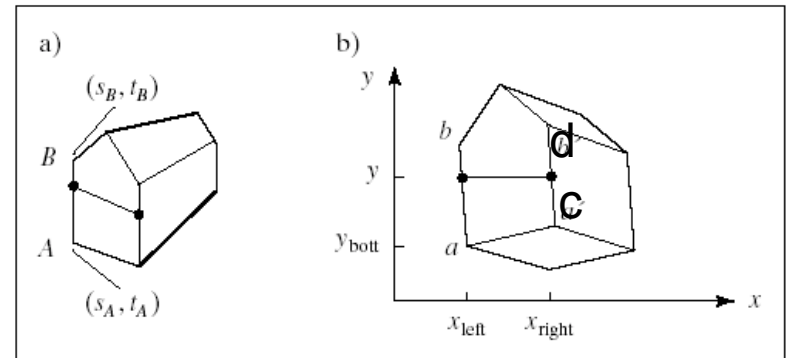
$$s_{left}(y) = \frac{\text{lerp}\left(\frac{s_A}{a_4}, \frac{s_B}{b_4}, f_l\right)}{\text{lerp}\left(\frac{1}{a_4}, \frac{1}{b_4}, f_l\right)}, s_{right}(y) = \frac{\text{lerp}\left(\frac{s_C}{c_4}, \frac{s_D}{d_4}, f_r\right)}{\text{lerp}\left(\frac{1}{c_4}, \frac{1}{d_4}, f_r\right)}$$

$$s(x, y) = \frac{\text{lerp}\left(\frac{s_{left}}{h_{left}}, \frac{s_{right}}{h_{right}}, f\right)}{\text{lerp}\left(\frac{1}{h_{left}}, \frac{1}{h_{right}}, f\right)}$$

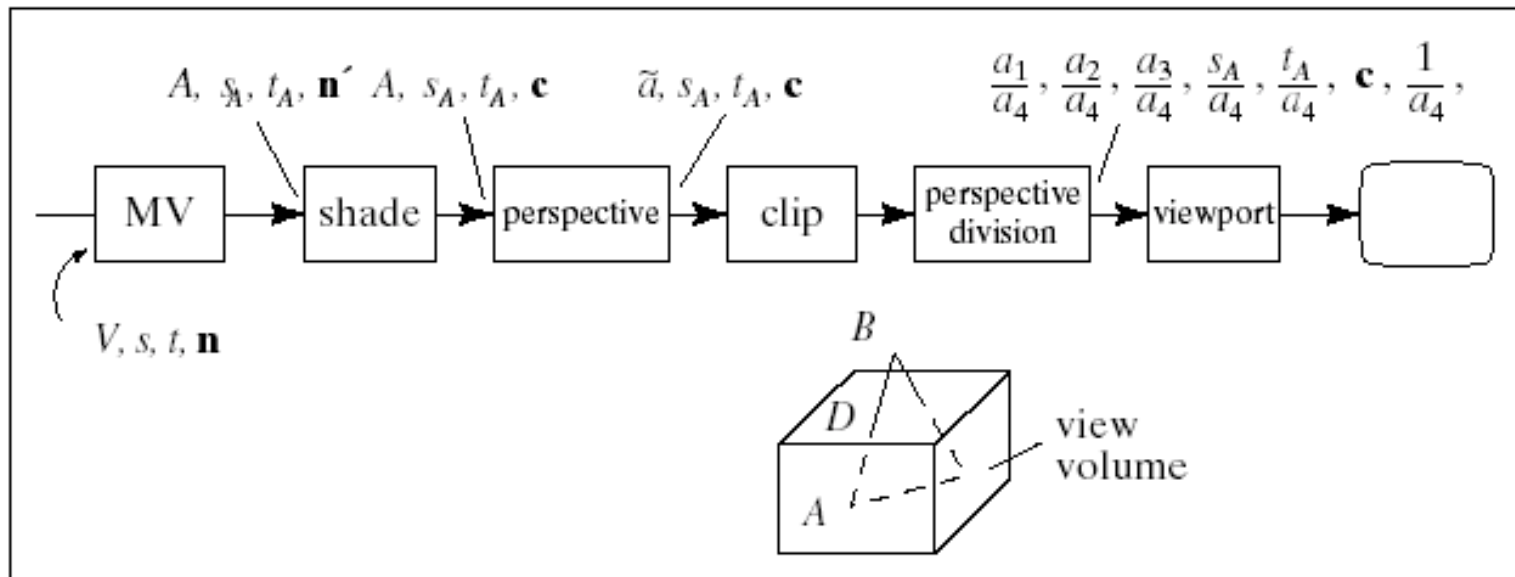
$$h_{left} = \text{lerp}(a_4, b_4, f_l)$$

$$h_{right} = \text{lerp}(c_4, d_4, f_r)$$

$$f = (x - x_{left}) / (x_{right} - x_{left})$$



Pipeline with hyperbolic interpolation



What does the texture do?

Replace

- $I_r = \text{texture}_r(s,t)$, similar for green and blue
- `glTexEnvf(GL_TEXTURE_ENV, GL_TEXTURE_ENV_MODE, GL_DECAL);`

Modulate

- $I = \text{texture}(s,t)[I_a k_a + I_d k_d \times \text{lambert}] + I_s k_s \times \text{phong}$
- `glTexEnvf(GL_TEXTURE_ENV, GL_TEXTURE_ENV_MODE, GL_MODULATE);`

Bump Mapping

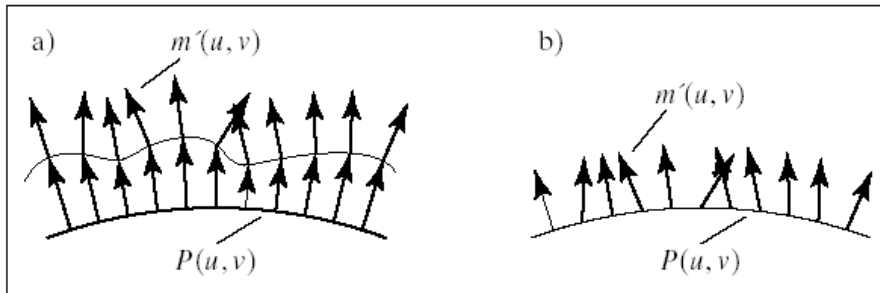


FIGURE 8.50 On the nature of bump mapping.

$$P'(u,v) = P(u,v) + \text{texture}(u,v)m(u,v)$$

Approximation by Blinn

$$m'(u,v) = m(u,v) + [(m \times P_v) \text{texture_u} - (m \times P_u) \text{texture_v}]$$

Where $_$ indicates partial derivative.

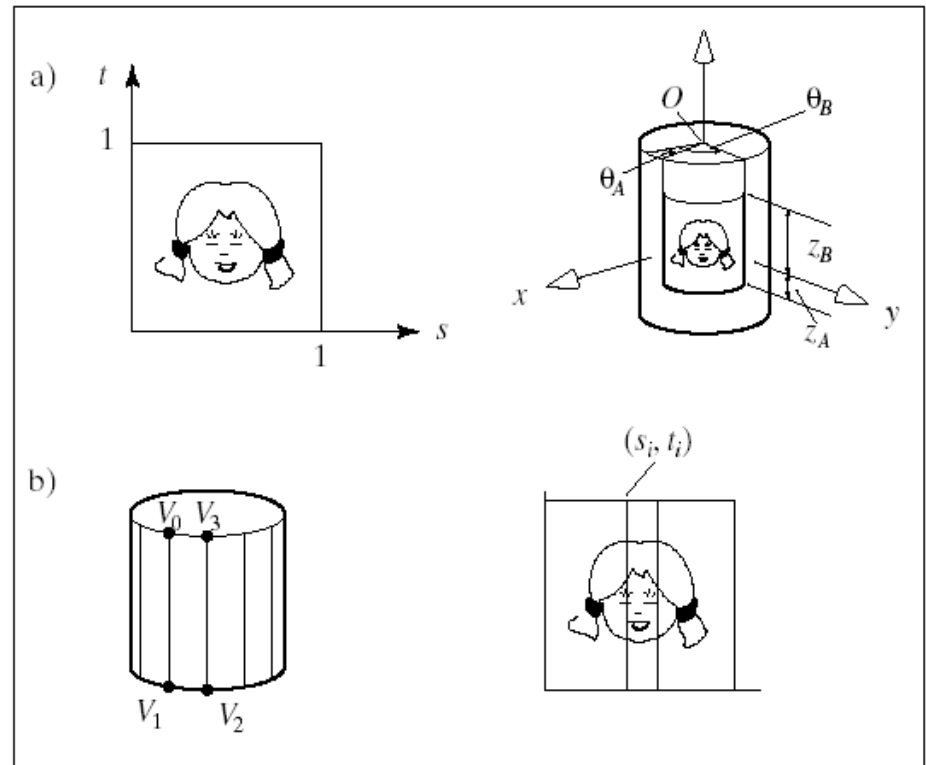
All functions evaluated at (u,v) .

Example



Calculating texture coordinates

Wrapping textures on curved surfaces



$$s = \frac{\theta - \theta_a}{\theta_b - \theta_a}, t = \frac{z - z_a}{z_b - z_a}$$

Cylinder with N faces

Left edge at azimuth $\theta = 2\pi i / N$

Upper left vertex texture coordinates $s_i = \frac{2\pi i / N - \theta_a}{\theta_b - \theta_a}, t_i = 1.$

Automatic calculation of Texture Coordinates

```
glEnable(GL_TEXTURE_GEN_S);
```

```
glEnable(GL_TEXTURE_GEN_T);
```

```
glTexGeni(GL_S, GL_TEXTURE_GEN_MODE,  
          GL_OBJECT_LINEAR);
```

```
GLfloat coeff[4] = {1,0,0,0} ;
```

```
glTexGenfv(GL_S, GL_OBJECT_PLANE, coeff);
```

Same for T

GL_OBJECT_LINEAR

Linear combination of coordinates

S or T = $p_0*x + p_1*y + p_2*z + p_4*w$

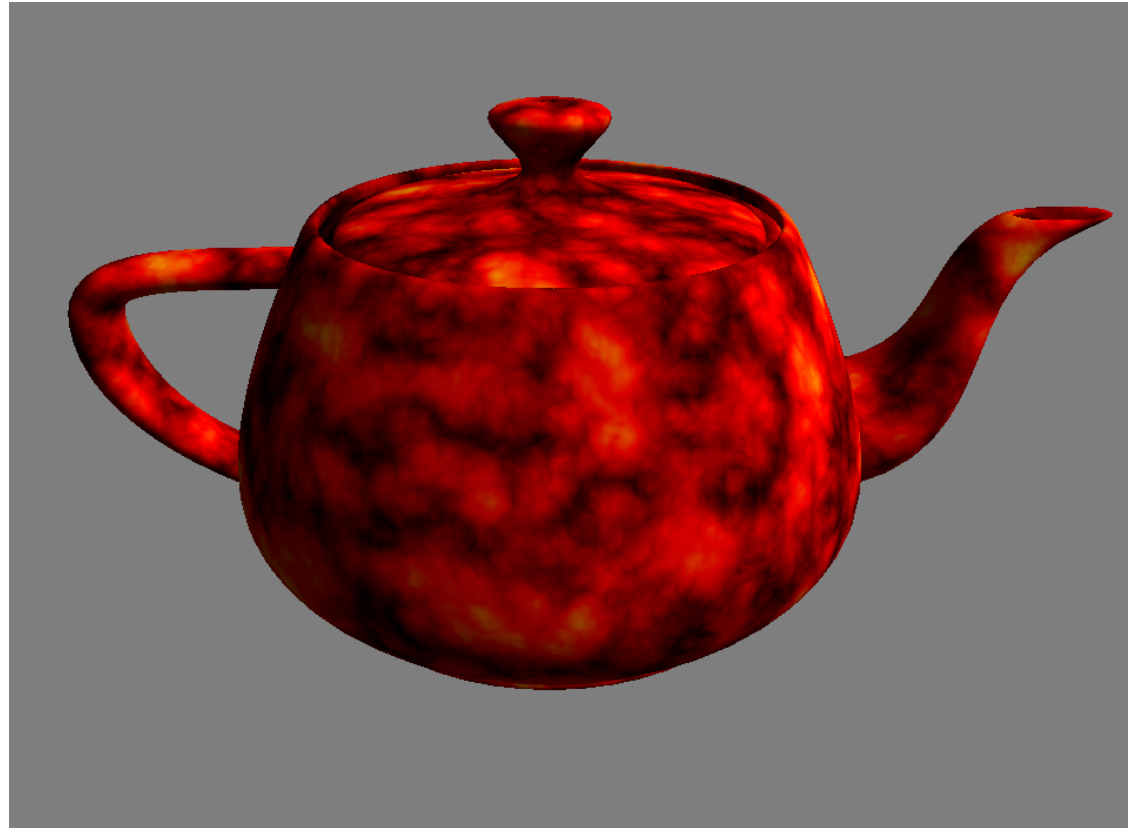
If (p_0, p_1, p_2, p_3) correctly normalized then
distance to plane (p_0, p_1, p_2, p_3)

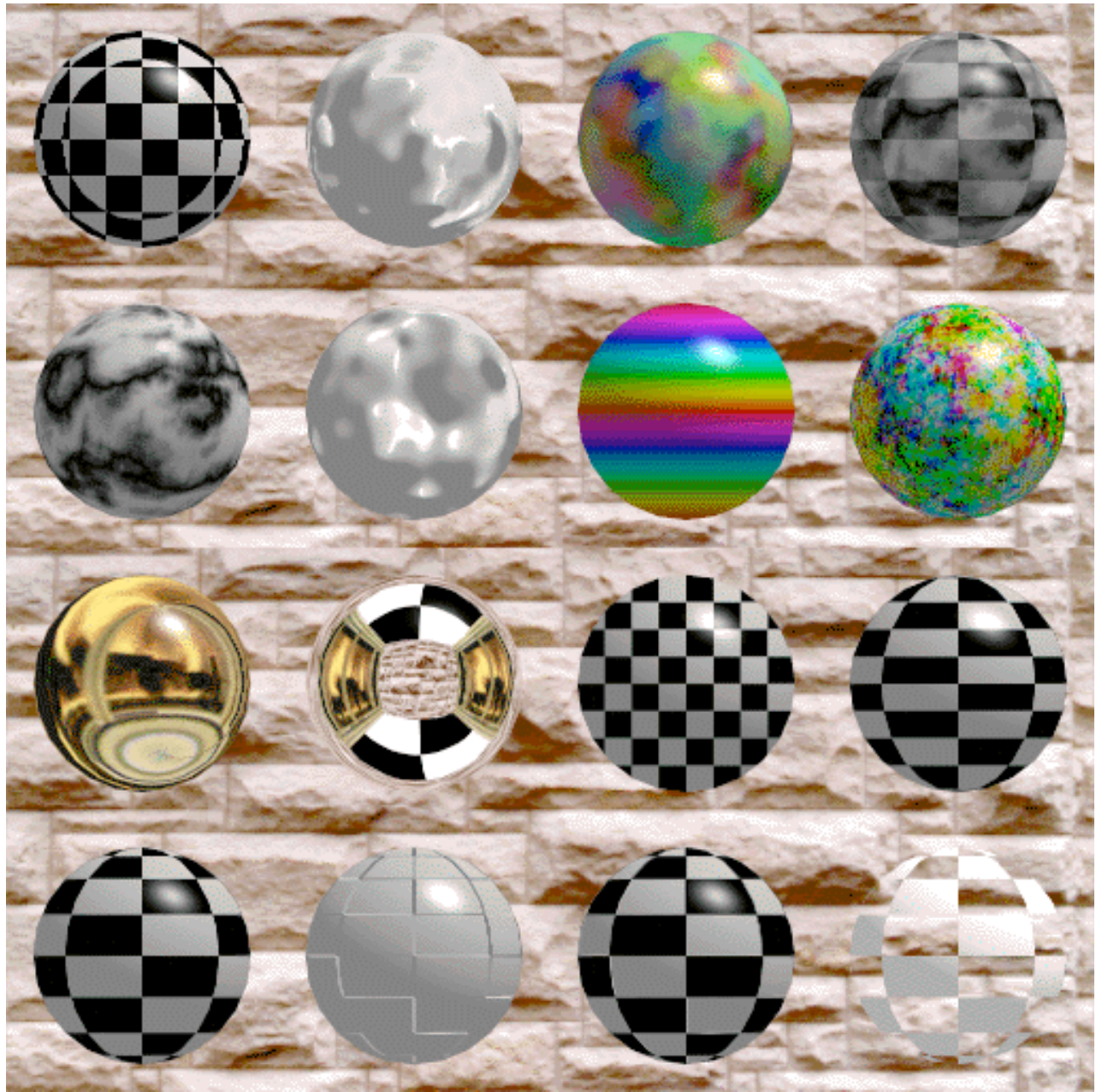
E.g. $(1, 0, 0, 0)$ distance from plane $x = 0$.

Procedural texture

Volumetric textures

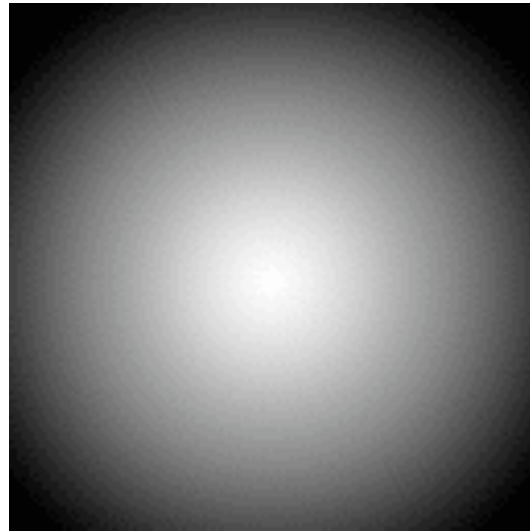
$$C = B(x,y,z)$$





Light maps

Static objects



Texture filtering

Texture images consist of pixels (texels).

Therefore:

- *Magnification:* a pixel on the screen may cover only part of a texel.
- *Minification:* a pixel on the screen may cover more than one texels.

Solution: Filtering

Texture filtering in OpenGL

```
glTexParameteri(GL_TEXTURE_2D,  
                GL_TEXTURE_MAG_FILTER, GL_NEAREST) ;  
glTexParameteri(GL_TEXTURE_2D,  
                GL_TEXTURE_MIN_FILTER, GL_NEAREST) ;
```

GL_TEXTURE_MAG_FILTER: GL_NEAREST or GL_LINEAR

GL_TEXTURE_MIN_FILTER: GL_NEAREST, GL_LINEAR,
 GL_NEAREST_MIPMAP_NEAREST,
 GL_LINEAR_MIPMAP_NEAREST,
 GL_LINEAR_MIPMAP_LINEAR,

Texture mapping in OpenGL

```
void glTexImage2D(  
    GLenum target,    // must be GL_TEXTURE_2D  
    GLint level,  
    GLint internalformat, // e.g. 3  
    GLsizei width,  
    GLsizei height,  
    GLint border,  
    GLenum format,    // e.g. GL_RGB  
    GLenum type,      // e.g. GL_UNSIGNED_BYTE  
    const GLvoid *pixels // size powers of 2 !!  
);
```

Texture Parameters

```
void glTexParameterf(  
    GLenum target,           // e.g. GL_TEXTURE_2D  
    GLenum pname,           // GL_WRAP_S  
    GLfloat param            // value e.g. GL_CLAMP  
);
```


Texture effect

```
void glTexEnvf(  
    GLenum target,      // GL_TEXTURE_ENV  
    GLenum pname,      // GL_TEXTURE_ENV_MODE  
    GLfloat param       // GL_MODULATE, GL_DECAL,  
                        // and GL_BLEND  
);
```

Enabling texture mapping

```
glEnable(GL_TEXTURE_2D) ;
```

```
glDisable(GL_TEXTURE_2D) ;
```

Texture mapping example

```
void initTexture(void) {
    glTexImage2D(GL_TEXTURE_2D, 0, 3, Img.m_width,
                Img.m_height, 0, GL_RGB, GL_UNSIGNED_BYTE,
                Img.m_data);
    glTexParameterf(GL_TEXTURE_2D, GL_TEXTURE_WRAP_S,
                    GL_REPEAT);
    glTexParameterf(GL_TEXTURE_2D, GL_TEXTURE_WRAP_T,
                    GL_REPEAT);
    glTexParameterf
    (GL_TEXTURE_2D, GL_TEXTURE_MAG_FILTER, GL_NEAREST);
    glTexParameterf(GL_TEXTURE_2D, GL_TEXTURE_MIN_FILTER,
                    GL_NEAREST);
    glTexEnvf(GL_TEXTURE_ENV, GL_TEXTURE_ENV_MODE,
              GL_DECAL);
    glEnable(GL_TEXTURE_2D);
}
```

Texture Objects

Copying an image from main memory to video memory is very expensive (glTexImage2D) .

- *Create texture names*
- *Bind (create) texture objects to texture data:*
 - *Image arrays + texture properties*
- *Bind and rebind texture objects.*

Naming texture objects

```
void glGenTextures (GLsizei n, GLuint  
    *textureNames) ;
```

Returns `n` unused names `textureNames[0]...[n]`).

```
GLboolean glIsTexture (GLuint textureName) ;
```

Creating Texture Objects

```
glBindTexture(GLenum target, GLuint  
textureName) ;
```

Three things:

- If textureName > 0 and not already assigned a new texture object is created.
- If textureName assigned, the textureObject becomes active.
- If textureName is 0 OpenGL stops using textures and returns to the default unnamed texture.

Initial creation

```
glBindTexture(GLenum target, GLuint  
textureName) ;
```

Target:

```
GL_TEXTURE_1D, GL_TEXTURE_2D , GL_TEXTURE_3D,  
GL_TEXTURE_CUBE_MAP.
```

Example: Creating the textures

```
void init(void) {
    glGenTextures(2, texName) ;
    glBindTexture(GL_TEXTURE_2D, texName[0]) ;
    glTexParameteri(GL_TEXTURE_2D, GL_TEXTURE_WRAP_S,
                    GL_CLAMP);
    glTexParameteri(GL_TEXTURE_2D, GL_TEXTURE_WRAP_T,
                    GL_CLAMP);
    glTexParameteri(GL_TEXTURE_2D, GL_TEXTURE_MAG_FILTER,
                    GL_NEAREST);
    glTexParameteri(GL_TEXTURE_2D, GL_TEXTURE_MIN_FILTER,
                    GL_NEAREST);
    glTexImage2D(GL_TEXTURE_2D, 0, GL_RGBA,
                GL_UNSIGNED_BYTE, image1) ;

    glBindTexture(GL_TEXTURE_2D, texName[1]) ;
    ...
    glTexImage2D(GL_TEXTURE_2D, 0, GL_RGBA,
                GL_UNSIGNED_BYTE, image2) ;
}
```


Example: Using the textures

```
void display(void) {  
    ..  
    glBindTexture(GL_TEXTURE_2D, texName[0]) ;  
    glBegin(GL_QUADS) ;  
    glTexCoord2f(0.0,0.0) ;  
  
    ..  
    glEnd() ;  
  
    glBindTexture(GL_TEXTURE_2D, texName[1]) ;  
    glBegin(GL_QUADS) ;  
    glTexCoord2f(0.0,0.0) ;  
  
    ..  
    glEnd() ;  
}
```