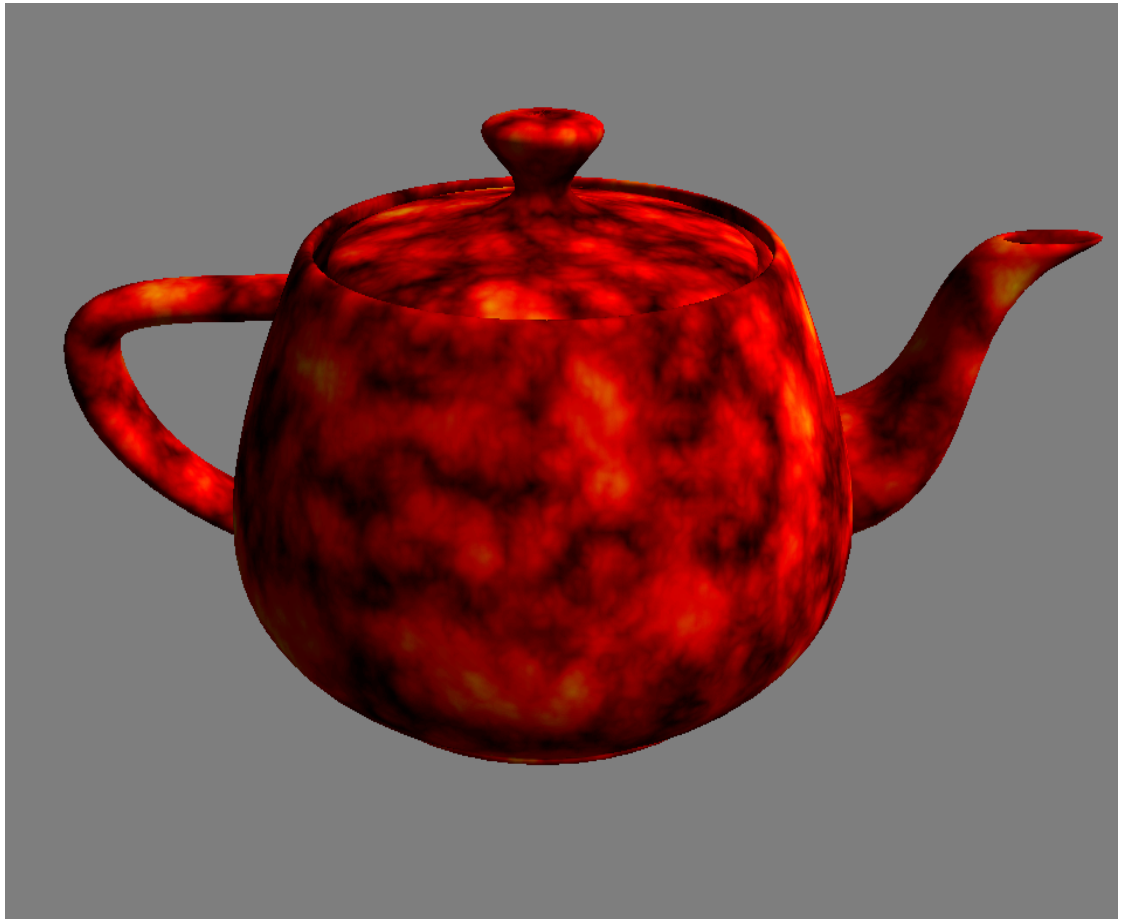
Surfaces



Formulations

Implicit: f(x,y,z) = 0

Normal $\mathbf{n} = \nabla(f)$

Explicit: z = f(x,y)

Parametric: x = f_x(s,t), y = f_y(s,t), z=f_z(s,t)

Quadric surfaces

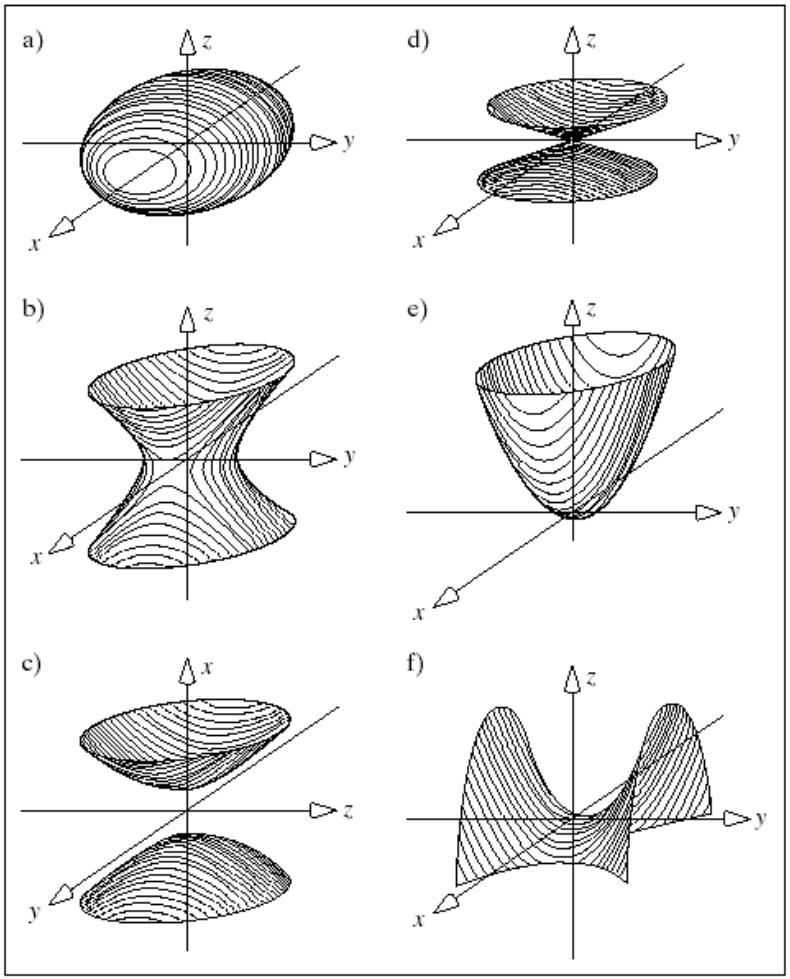


FIGURE 6.70 The six quadric surfaces: (a) Ellipsoid. (b) Hyperboloid of one sheet. (c) Hyperboloid of two sheets. (d) Elliptic cone. (e) Elliptic paraboloid. (f) Hyperbolic paraboloid.

Quadric surfaces

Sphere:

$$f(x,y,z) = (x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 - R^2 = 0$$

$$x(\phi,\theta) = Rcos(\phi)cos(\theta)$$
$$y(\phi,\theta) = Rcos(\phi)sin(\theta)$$
$$z(\phi,\theta) = Rsin(\phi)$$
$$-\pi/2 \leq \phi \leq \pi/2$$
$$-\pi \leq \theta \leq \pi$$

Quadric surfaces

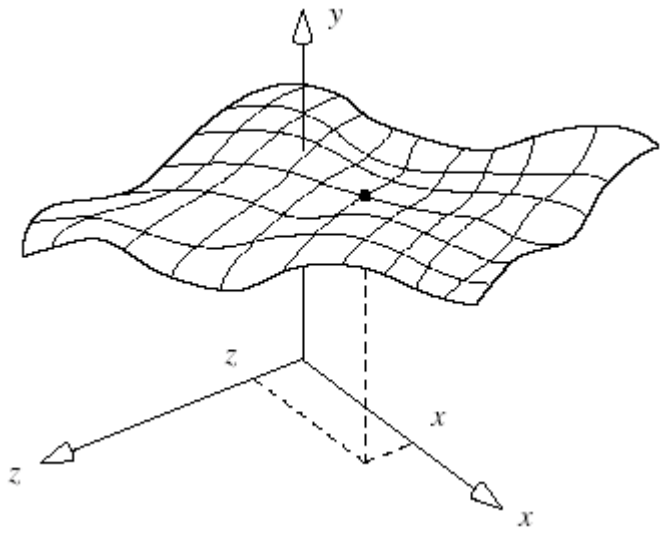
Ellipsoid

$$f(x,y,z) = (\frac{x-x_0}{R_x})^2 + (\frac{y-y_0}{R_y})^2 + (\frac{z-z_0}{R_z})^2 - 1 = 0$$

$$x(\phi,\theta) = R_x cos(\phi) cos(\theta)$$
$$y(\phi,\theta) = R_y cos(\phi) sin(\theta)$$
$$z(\phi,\theta) = R_z sin(\phi)$$
$$-\pi/2 \leq \phi \leq \pi/2$$
$$-\pi \leq \theta \leq \pi$$

Height fields

y=f(x,z)

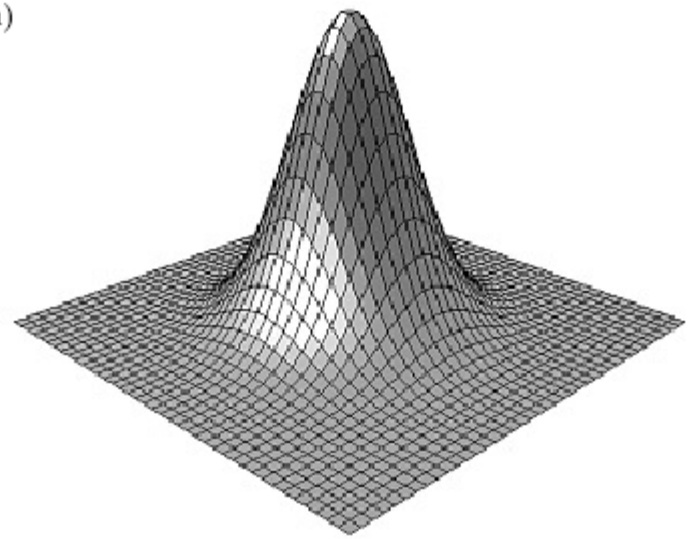


Typical height fields

Gaussian

$$y = f(x, z) = e^{-ax^2 - bz^2}$$

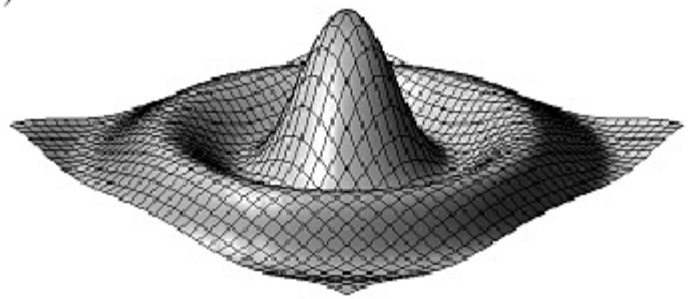
a)



Sinc

$$y = f(x, z) = \frac{sin(\sqrt{x^2 + z^2})}{\sqrt{x^2 + z^2}}$$

b)



Parametric formulations

Ruled surfaces:

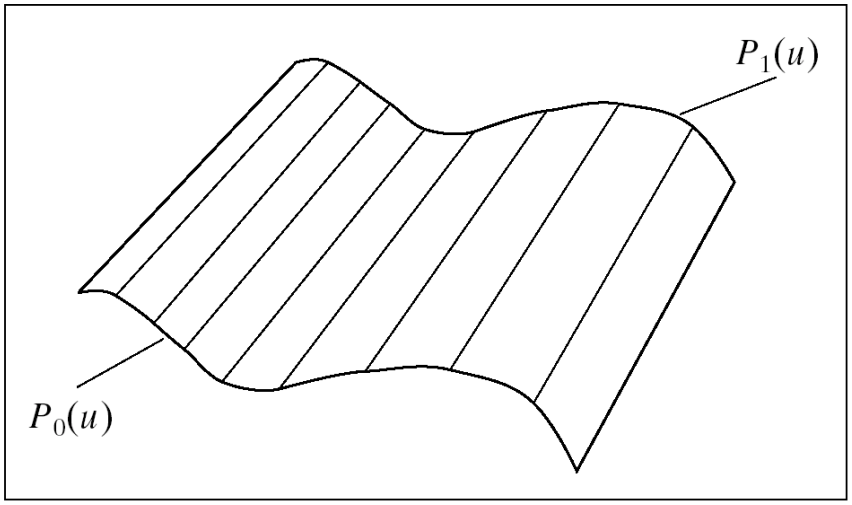
Linear combination of two curves

- Through every point on the surface there passes at least one line that lies on the surface

$$P(u) = (1-u)P_0 + uP_1$$

Making P_0 and P_1 curves:

$$P(u,v) = (1-u)P_0(v) + uP_1(v)$$

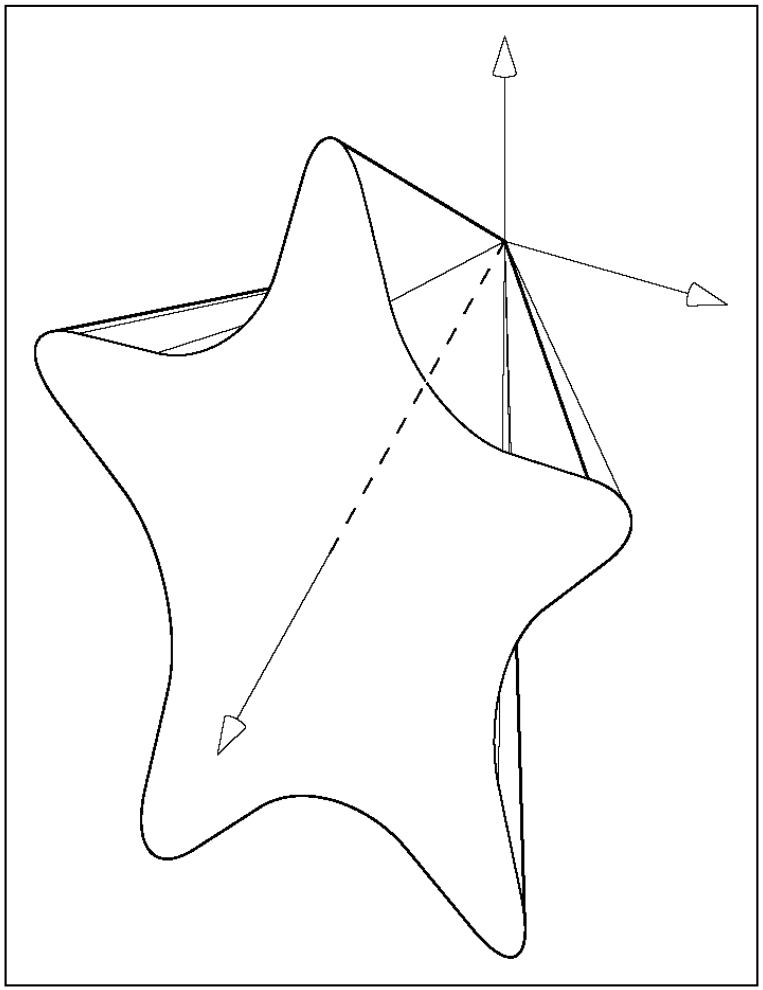


Special cases

General cone

$$P(u,v) = (1-u)P_0 + uP_1(v)$$

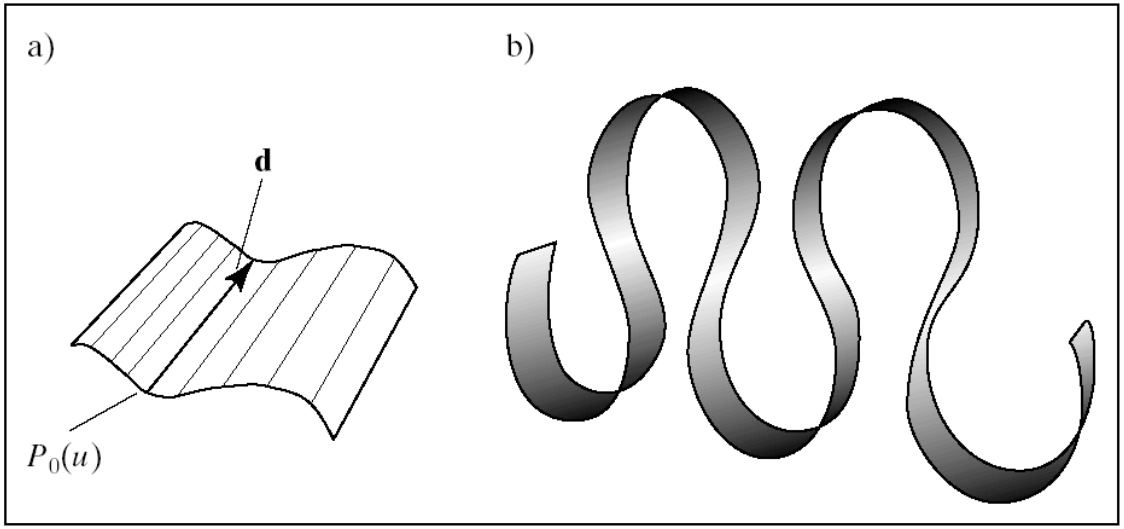
P_0 is the apex



General Cylinder

P1 a translated version of P0

$$P(u,v) = (1-u)P_0(v) + u(P_0(v) + \mathbf{d}) \Rightarrow P(u,v) = P_0(v) + u\mathbf{d}$$



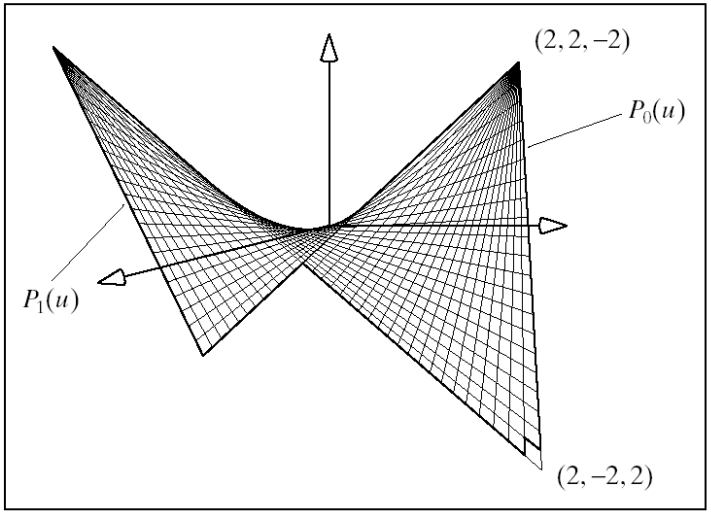
Bilinear patches

Both P1 and P0 are lines

$$P(u,v) = (1-u)P_0(v) + uP_1(v) \Rightarrow$$

$$P(u,v) = (1-u)[(1-v)P_{00} + vP_{01}] + u[(1-v)P_{10} + vP_{11}] \Rightarrow$$

$$P(u,v) = (1-u)(1-v)P_{00} + (1-u)vP_{01} + u(1-v)P_{10} + uvP_{11}$$

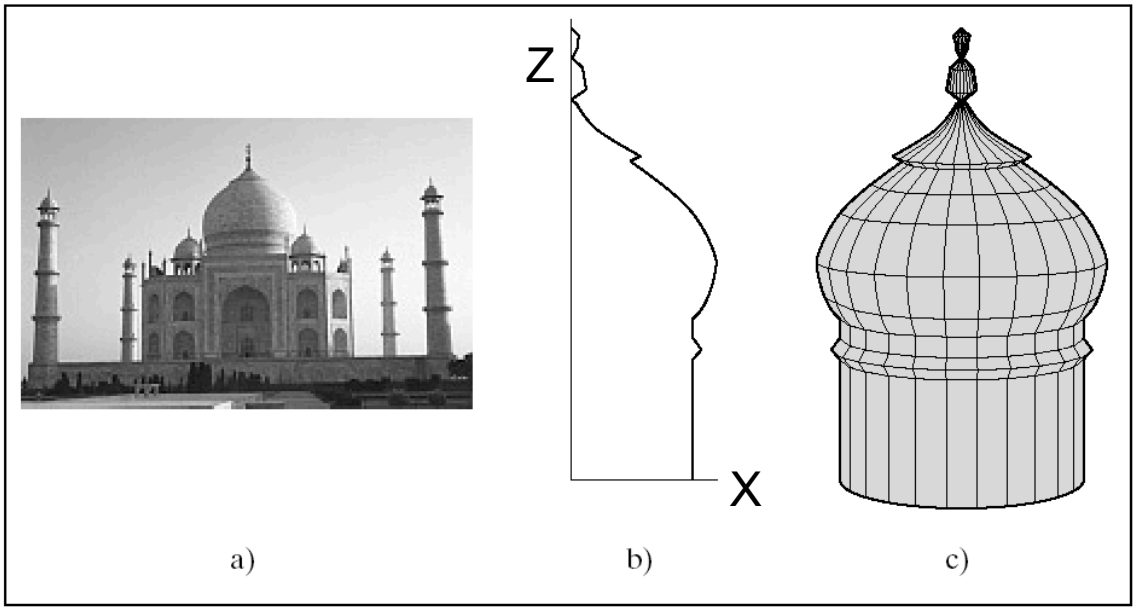


Surfaces of revolution

Sweep profile curve around an axis:

C(v) = (X(v),Z(v))

P(u,v)=(X(v)cos(u),X(v)sin(u), Z(v))



Parametric surfaces from control points (constraints)

Extension of the curve form to two dimensions

Curve: $X(s) = SMG = [s^3\ s^2\ s\ 1]MG$

Surface: $X(s,t) = SMG(t)$

Bezier Surfaces

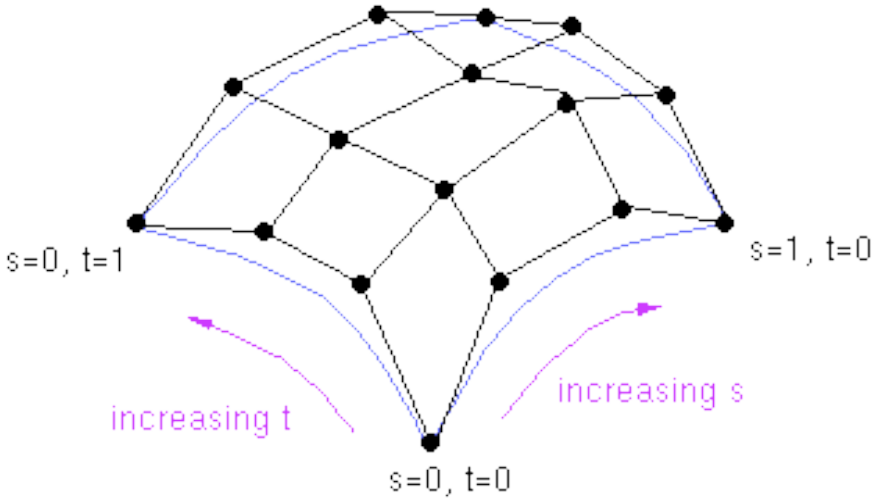
Take a bezier curve P(s) and let its control points become bezier curves

$P(s) = S\ M\ G$

$G(t) = [P_1(t)\ P_2(t)\ P_3(t)\ P_4(t)]^T$

Where

$P_i(t) = T\ M\ G_i$



Compact representation

Tensor product

$$P1(t) = \mathbf{G_1}^T \mathbf{M}^T \mathbf{T}^T$$

$$P2(t) = \mathbf{G_2}^T \mathbf{M}^T \mathbf{T}^T$$

$$P3(t) = \mathbf{G_3}^T \mathbf{M}^T \mathbf{T}^T$$

$$P4(t) = \mathbf{G_4}^T \mathbf{M}^T \mathbf{T}^T$$

$$P(s,t) = \mathbf{SMG} = \mathbf{SM}\begin{bmatrix} P1(t) \\ P2(t) \\ P3(t) \\ P4(t) \end{bmatrix} = \mathbf{SM}\begin{bmatrix} \mathbf{G_1}^T \mathbf{M}^T \mathbf{T}^T \\ \mathbf{G_2}^T \mathbf{M}^T \mathbf{T}^T \\ \mathbf{G_3}^T \mathbf{M}^T \mathbf{T}^T \\ \mathbf{G_4}^T \mathbf{M}^T \mathbf{T}^T \end{bmatrix} = \mathbf{SM}\begin{bmatrix} g11 & g12 & g13 & g14 \\ g21 & g22 & g23 & g24 \\ g31 & g32 & g33 & g34 \\ g41 & g42 & g43 & g44 \end{bmatrix}\mathbf{M}^T \mathbf{T}^T$$

$$x(s,t) = \mathbf{SMG}_x \mathbf{M}^T \mathbf{T}^T$$

$$y(s,t) = \mathbf{SMG}_y \mathbf{M}^T \mathbf{T}^T$$

$$z(s,t) = \mathbf{SMG}_z \mathbf{M}^T \mathbf{T}^T$$

Properties of Bezier surfaces

Affine Invariance

Convex Hull

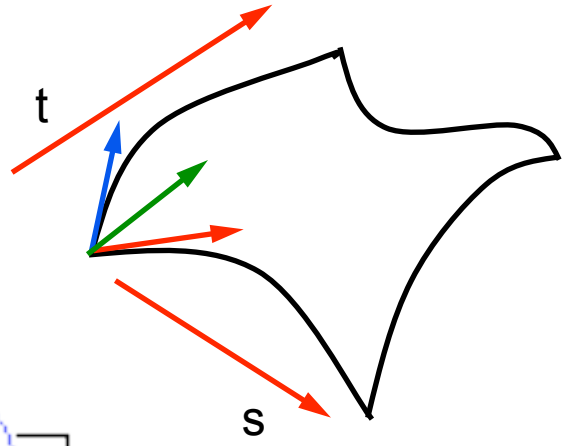
Plane precision

Variation diminishing

Hermite surfaces

Constraints at the four corners:

- Position, Tangent, Twist



$$G_x = \begin{bmatrix} x(0,0) & x(0,1) & \frac{\partial}{\partial t}x(0,0) & \frac{\partial}{\partial t}x(0,1) \\ x(1,0) & x(1,1) & \frac{\partial}{\partial t}x(1,0) & \frac{\partial}{\partial t}x(1,1) \\ \frac{\partial}{\partial s}x(0,0) & \frac{\partial}{\partial s}x(0,1) & \frac{\partial^2}{\partial s \partial t}x(0,0) & \frac{\partial^2}{\partial s \partial t}x(0,1) \\ \frac{\partial}{\partial s}x(1,0) & \frac{\partial}{\partial s}x(1,1) & \frac{\partial^2}{\partial s \partial t}x(1,0) & \frac{\partial^2}{\partial s \partial t}x(1,1) \end{bmatrix}$$

points

tangents

tangents

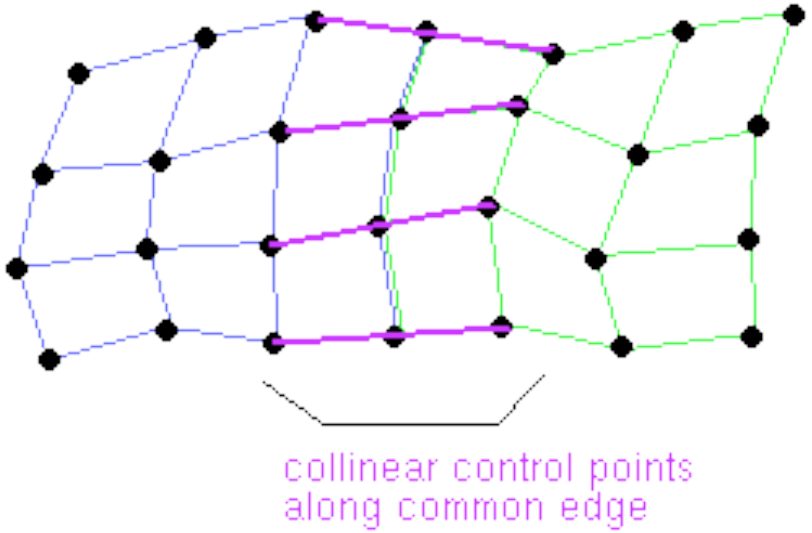
twist vectors

Piecewise cubic bezier surfaces

G1 continuity

Common edge

Make 2 sets of 4 control points collinear



Rendering parametric curves and surfaces

Transform into primitives we know how to handle

Curves

- Line segments

Surfaces

- Quadrilaterals
- Triangles

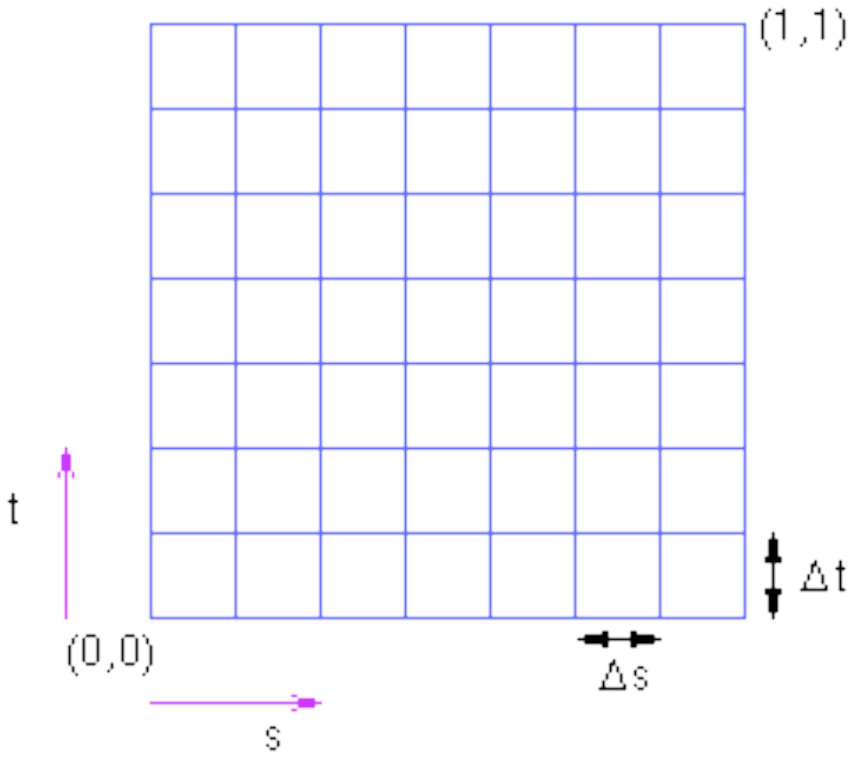
Converting to quadrilaterals

Straightforward

Uniform subdivision

Evaluation of P(s,t) at each grid point

Isoparametric lines become isoparametric curves



Optimizations

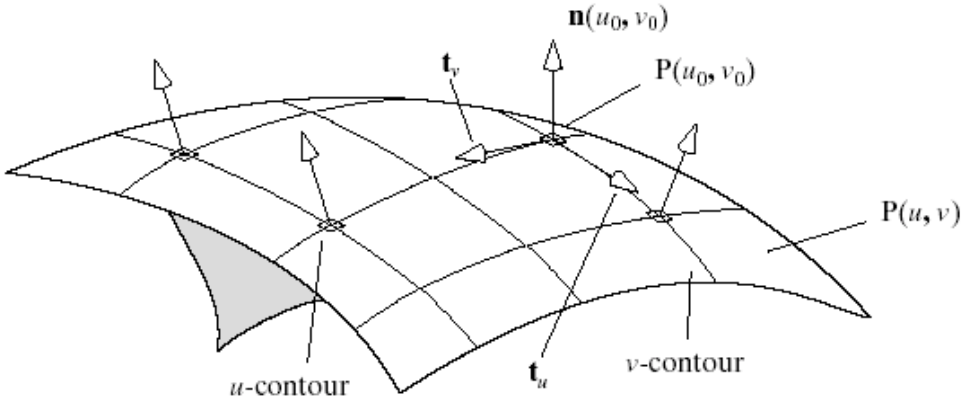
$x(s,t) = S\ M\ G\ M^T\ T^T$

- $M\ G\ M^T$ remains constant over patch: precompute
- $S\ M$ and $M^T\ T^T$ remain constant over all patches:
  precompute $S\ M$ and store in $Q[s]$
  $Q[t] = Q^T[s]$ assuming equal subdivisions in s and t

Computing surface normals

Parametric surface P(u,v)

P(u,v) = U M G M^T V^T



$$\mathbf{N} = \frac{\partial P(u,v)}{\partial u} \times \frac{\partial P(u,v)}{\partial v}$$

Surfaces in OpenGL

Two dimensional evaluators

void glMap2f(GLenum target, GLdouble u1, GLdouble u2, GLint ustride, GLint uorder, GLdouble v1, GLdouble v2, GLint vstride, GLint vorder, const GLfloat *points)

void **glEvalCoord2dv**(const GLdouble **u*)

void **glEvalCoord2fv**(const GLfloat **u*)

Grid approach

void glMapGrid2f(GLint un, GLfloat u1, GLfloat u2, GLint vn, GLfloat v1, GLfloat v2)

Textures

void glMap2f(GLenum target, GLdouble u1, GLdouble u2, GLint ustride, GLint uorder, GLdouble v1, GLdouble v2, GLint vstride, GLint vorder, const GLfloat *points)

target = GL_MAP2_TEXTURE_2