University of California, Los Angeles May 4, 2018
CS 289 Communication Complexity
Instructor: Alexander Sherstov

Instructions

- Your solutions are due at 11:59 pm on May 13, 2018. Please typeset your work and submit the resulting PDF document on CCLE using the TurnItIn feature. Late work will not be accepted or graded, resulting in zero credit for this assignment.

- You cannot collaborate with other students on this assignment. The work that you submit must be your own.

- This exam can be solved in its entirety using the course material taught so far, without consulting any additional sources. However, you are welcome to use any scholarly sources, including your textbook and the Internet. If you do, please write the solution in your own words and acknowledge the sources that you have consulted.

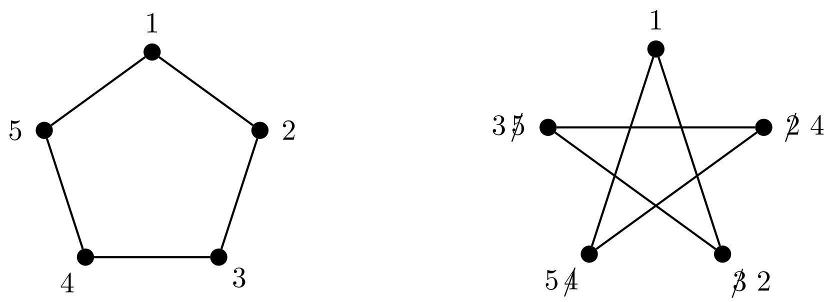
- If you are using a fact that we have not covered in class, please provide a proof for it. This applies even to facts that are published and well-known.

- If you are not able to solve a problem in full, make a simplifying assumption. Start early, do your best, and take pride in your discoveries.

Most importantly, have fun!

Midterm Exam

1 ISOMORPHIC GRAPHS.
Two undirected graphs on n vertices are *isomorphic* if one results from the other by renumbering the vertices, as in the figure below. How much communication do Alice and Bob need to find out deterministically if their graphs are isomorphic?



2 INTEGER MULTIPLICATION.
Alice and Bob's inputs are integers a and b, respectively, where $a, b \in [0, 2^n - 1]$. Prove that computing the n^{th} bit of (the binary representation of) the product $a \cdot b$ requires $\Omega(n/\log n)$ bits of communication.

3 INNER PRODUCT MODULO 18181.
Define $f\colon \{0,1\}^n \times \{0,1\}^n \to \{0,1\}$ by $f(x,y) = 1$ iff $\sum x_i y_i \equiv 0 \pmod{18181}$. Prove that f has no fooling set larger than n^c, for some constant c.

4 INNER PRODUCT MODULO 18181, AGAIN.
What is the nondeterministic communication complexity of f in the previous problem?

5 ORTHOGONAL SUBSPACES.
On input linear subspaces $A, B \subseteq \mathbb{F}_2^n$, prove that $\Theta(n^2)$ bits of nondeterministic communication are necessary and sufficient to check if A and B are orthogonal.

6 A COMMUNICATION-RANDOMNESS TRADE-OFF.
Prove that any randomized protocol for EQ_n with probability of correctness $2/3$ and communication cost c must use more than $\log_2(n/c)$ bits of randomness.

7 BETTER THAN RANDOM.
Prove that every communication problem $f\colon \{0,1\}^n \times \{0,1\}^n \to \{0,1\}$ has a randomized protocol with constant cost and error at most $\frac{1}{2} - \Theta(2^{-n/2})$.