

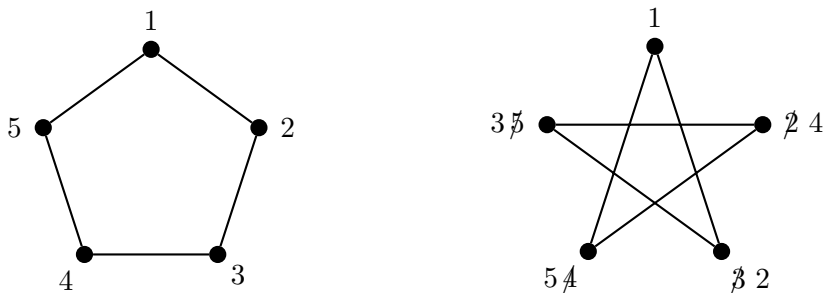
Instructions

- Your solutions are due next Wednesday, May 4 at the beginning of class. Late work will not be accepted or graded, resulting in zero credit for this assignment.
- Please typeset your solutions and hand in a printed copy.
- You should think of this assignment as a take-home exam. Specifically, you cannot discuss this problem set with your fellow classmates or anyone else.
- In addition to the textbook and lecture notes, feel free to use any scholarly sources such as published research and the Internet. Please remember to acknowledge any sources that you have consulted.
- If unable to solve a problem in full, try to derive a weaker bound or solve the problem under an unproven simplifying assumption.

Most importantly, have fun!

Midterm Exam

- Two undirected graphs on n vertices are *isomorphic* if one results from the other by renumbering the vertices, as in the figure below. How much communication do Alice and Bob need to find out deterministically if their graphs are isomorphic?



- Construct a problem $\{f_n\}_{n=1}^{\infty}$ that is complete for the communication class P.
- Define $f: \{0,1\}^n \times \{0,1\}^n \rightarrow \{0,1\}$ by $f(x,y) = 1$ iff $\sum x_i y_i \equiv 0 \pmod{18181}$. Prove that f has no fooling set larger than n^c , for some constant c .
- What is the nondeterministic communication complexity of f in the previous problem?
- On input linear subspaces $A, B \subseteq \mathbb{F}_2^n$, prove that $\Theta(n^2)$ bits of nondeterministic communication are necessary and sufficient to check if A and B are orthogonal.
- Prove that every communication problem $f: \{0,1\}^n \times \{0,1\}^n \rightarrow \{0,1\}$ has a randomized protocol with constant cost and error at most $\frac{1}{2} - \Theta(2^{-n/2})$.