University of California, Los Angeles — June 1, 2016
CS 289 Communication Complexity
Instructor: Alexander Sherstov

Instructions

- Your solutions are due at 11:59pm next Friday, June 10. Please typeset your work and submit the resulting PDF document on myUCLA using the TurnItIn feature. Late work will not be accepted or graded, resulting in zero credit for this assignment.

- You cannot collaborate with other students on this assignment. The work that you submit must be your own.

- This exam can be solved in its entirety using the course material taught so far, without consulting any additional sources. However, you are welcome to use any scholarly sources, including your textbook and the Internet. If you do, please write the solution in your own words and acknowledge the sources that you have consulted.

- If unable to solve a problem in full, make a simplifying assumption. Start early, do your best, and take pride in your discoveries.

Most importantly, have fun!

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Final Exam

1 Let M be a real matrix. You may, at any time, select a row or column and flip the signs of all the entries in it. Prove that there is a sequence of sign flips that results in a matrix in which all row sums and all column sums are nonnegative.

2 Prove that $\mathrm{disc}(\mathrm{DISJ}_n) \leqslant O(1/\sqrt{n})$. *Hint:* there is a very short proof that requires no calculations!

3 Prove that $R_{1/3}(\mathrm{IP}_n) \geqslant n - O(1)$.

4 Use the approach of convex relaxations to derive the *generalized discrepancy method*: for any $\varphi \not\equiv 0$,

$$2^{R_\varepsilon(f)} \geqslant \frac{\langle (-1)^f, \varphi\rangle - 2\varepsilon\|\varphi\|_1}{\max_{\text{rectangle } R} |\langle R, \varphi\rangle|}.$$

5 Prove that the choice of error parameter $0 < \varepsilon < 1/2$ affects the approximate degree of a Boolean function by at most a multiplicative constant, i.e., for any $\varepsilon, \delta \in (0, 1/2)$, we have $c_{\varepsilon,\delta} \deg_\varepsilon(f) \leqslant \deg_\delta(f) \leqslant C_{\varepsilon,\delta} \deg_\varepsilon(f)$ for some $C_{\varepsilon,\delta} > c_{\varepsilon,\delta} > 0$ and all Boolean functions f.

6 (*Communicated by W. Rosenbaum*) A group of n prisoners are brought into a room with n chairs arranged in a circle. Once they are all seated, each prisoner has a hat placed on their head. There are n distinct colors of hats, which are known to the prisoners beforehand, though colors may be repeated or not appear on any prisoner's head. Each prisoner can see everyone's hat color except for their own. The prisoners must then simultaneously call out a guess for their own hat color. The prisoners are set free if at least one prisoner correctly guesses their own hat color, and are all executed otherwise. Is it possible for the prisoners to guarantee that they will all be set free? The prisoners can devise a strategy before entering the room, but cannot communicate once in the room.