

## Instructions for Problem Set I

- Your solutions are due November 13 at the beginning of class. Late work will not be accepted or graded, resulting in zero credit for this assignment.
- Please typeset your solutions and hand in a printed copy.
- You cannot collaborate with other students on this assignment. The work that you submit must be your own.
- In addition to the textbook and lecture notes, feel free to use any outside sources such as the research literature and Internet materials. Please remember to acknowledge any sources that you have consulted.
- If unable to solve a problem in full, try to derive a weaker bound or solve the problem under an unproven simplifying assumption.

Most important, have fun!

## Problem Set I

- 1 Let  $f: \{0, 1\}^n \times \{0, 1\}^n \rightarrow \{0, 1\}$  be given by  $f(x, y) = 1$  iff  $\sum x_i y_i \equiv 0 \pmod{2011}$ . Prove that  $f$  has no fooling set larger than  $n^c$ , for some constant  $c$ .
- 2 What is the nondeterministic communication complexity of the function  $f$  above?
- 3 Two undirected graphs on  $n$  vertices are *isomorphic* if one results from the other by renumbering the vertices, as below. How much communication do Alice and Bob need to find out deterministically if their graphs are isomorphic?



- 4 On input linear subspaces  $A, B \subseteq \mathbb{F}_2^n$ , prove that  $\Theta(n^2)$  bits of nondeterministic communication are necessary and sufficient to check if  $A$  and  $B$  are orthogonal.
- 5 Alice and Bob each have a matrix, over some finite field. How much communication do they need to determine with accuracy 99% if their matrices are inverses of each other?
- 6 Let  $M_1, M_2, \dots, M_k$  be Boolean matrices with cover numbers  $c_1, c_2, \dots, c_k$ , respectively. Determine the cover number for

$$\begin{bmatrix} M_1 & & & \mathbf{0} \\ & M_2 & & \\ & & \ddots & \\ \mathbf{0} & & & M_k \end{bmatrix}.$$