CS 565 May 7, 1998

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Final Exam + Qualifying Exam

Submit your solutions on separate paper (don't submit this handout). Write your name and id number on the top of the first page you submit. I will stable your pages together. This exam is intended to take you 2 hours. Budget your time wisely. There are 4 questions on the exam. Each question is worth 5 points, for a total of 20 points maximum. Write the solutions for different questions on different sheets of paper. Make clear what problem each solution is for. I have spare paper, just ask for it.

To those who are taking the qualifying exam: to pass the exam you must get an excellent score.

1. [**Flow Analysis of Simply-Typed Programs**] This question is about extending the type rules for simply-typed λ-calculus such that they also do flow analysis. We will consider the language of pure λ-terms where each λ-abstraction has a unique label from a label set L:

$$e ::= x \mid \lambda^l x.e \mid e_1 e_2 \ ,$$

and we will consider the types

$$t ::= \mathsf{Int} \mid t_1 \xrightarrow{s} t_2$$

where $s \subseteq L$. The idea is that if

$$A \vdash e : (t_1 \xrightarrow{s} t_2)$$

is derivable, then s is flow information for e, and if we strip away the label-sets from A and $(t_1 \xrightarrow{s} t_2)$, then we get an ordinary type environment and a simple type for e. Define type rules for the types above, and justify informally why the flow-analysis aspects of the type rules are sound.

2. [**Structural Subtyping**] Here are two types s, t:

$$\begin{aligned} s &= \mu\alpha.((\alpha \to \alpha) \to \bot) \\ t &= \mu\beta.(((\beta \to (\top \to \bot)) \to \bot) \to (\mu\gamma.(\gamma \to \bot))) \end{aligned}$$

Is $s \leq t$ in the Amadio/Cardelli type ordering? Justify your answer.

3. [**Simple Types**] Consider the following mapping φ from pure λ-terms to pure λ-terms:

$$\begin{array}{rcl}\varphi(x) & = & \lambda k.kx \\ \varphi(\lambda x.e) & = & \lambda k.k(\lambda x.\varphi(e)) \\ \varphi(e_1e_2) & = & \lambda k.(\varphi(e_1))(\lambda v_1.(\varphi(e_2))(\lambda v_2.(v_1v_2)k))\end{array}$$

Consider the type system for pure λ-terms where types are generated from the grammar

$$t ::= \mathsf{Int} \mid t \to t$$

and the type rules are the usual ones for simple types. It is a known theorem about φ that there exist a mapping ψ from types to types, and a mapping Γ from type environments to type environments such that if A is a type environment, e is a λ-term, t is a type, and $A \vdash e : t$ is derivable, then $\Gamma(A) \vdash \varphi(e) : \psi(t)$ is derivable. Define such a ψ and such a Γ, and justify informally why they satisfy the theorem.

4. [**Fast Control-Flow Analysis**] Consider the following lambda-term:

$$G = (\lambda^1 f.(f(\lambda^2 x.x))(ff))(\lambda^3 y.y)$$

Show the graph for G that is used by the Heintze/McAllester $O(n^2)$ time flow analysis algorithm. What is the label set for G produced by the algorithm?