CS 565, Jens Palsberg May 2, 2002

Final Exam + Qualifying Exam

Each of the four questions is worth five points, for a total of 20 points. Write your name and id number on the first page you submit. Write solutions for different questions on different sheets of paper. Don't submit this handout.

1. [**Escape Analysis**] Consider a subset of Java in which classes are defined by the following grammar:

$$
\begin{array}{rcl}
\mathit{ClassDecl} & ::= & \textbf{class}\ \mathit{id}\ \{\ \mathit{VarDecl}^*\ \ \mathit{MethodDecl}^*\ \} \\
\mathit{VarDecl} & ::= & \mathit{Type}\ \mathit{id}\ ; \\
\mathit{MethodDecl} & ::= & \textbf{public void}\ \mathit{id}\ (\ \mathit{FormalList}\ ) \\
 & & \quad \{\ \mathit{VarDecl}^*\ \ \mathit{Statement}^*\ \} \\
\mathit{Statement} & ::= & \textbf{if}\ (\ \mathit{Exp}\ )\ \{\ \mathit{Statement}^*\ \}\ \textbf{else}\ \{\ \mathit{Statement}^*\ \} \\
 & ::= & \mathit{id}\ \texttt{=}\ \mathit{Exp}\ ; \\
 & ::= & \mathit{Exp}\ .\ \mathit{id}\ (\ \mathit{ExpList}\ ) \\
\mathit{Exp} & ::= & \textbf{true} \\
 & ::= & \textbf{false} \\
 & ::= & \mathit{id} \\
 & ::= & \textbf{this} \\
 & ::= & \textbf{new}\ \mathit{id}\ ()
\end{array}
$$

We say that an object *escapes* a method in case 1) the object is created during a method call and 2) the object is still reachable after the method call. Present a static analysis which for a given occurrence of **new** *id* () conservatively estimates whether the created object can escape the method. Here "conservative" means that the analysis should answer either "no" or "don't know."

2. [**Fast Control-Flow Analysis**] Consider the following lambda-term:

$$G = (\lambda^1 f.f(\lambda^2 a.\lambda^3 b.b))(\lambda^4 v.vy)$$

Show the graph for G that is used by the Heintze/McAllester $O(n^2)$ time flow analysis algorithm. What is the label set for G produced by the algorithm?

3. [Type Preservation] Consider the λ-terms and the types generated by the grammars:

$$\begin{array}{lcl} e & ::= & x \mid \lambda x.e \mid e_1 e_2 \\ t & ::= & t \to t \mid \alpha. \end{array}$$

Assume the usual type rules for simply-typed λ-calculus. The function $\mathcal{E}$ takes a type and a λ-term, and it produces a λ-term:

$$\begin{array}{rcl} \mathcal{E}^{\alpha}(e) & = & e \\ \mathcal{E}^{t_1 \to t_2}(e) & = & \lambda x.\mathcal{E}^{t_2}(e(\mathcal{E}^{t_1}(x))) \end{array}$$

Prove that if $A \vdash e : t$, then $A \vdash \mathcal{E}^t(e) : t$.

4. [Subtyping] Here are grammars for a set of λ-terms and for a set of types:

$$\begin{array}{lcl} n & ::= & \lambda x.n \mid x n_1 \dots\ n_k \qquad k \geq 0 \\ t & ::= & t \to t \mid \top. \end{array}$$

An expression $xn_1 \dots\ n_k$ should be read as $(((xn_1)n_2)\dots)n_k$. Notice that every n generated by the grammar is in β normal form; no β-reduction steps are possible. Consider the following type rules:

$$A \vdash x : t \quad (A(x) = t) \tag{1}$$

$$\frac{A[x : s] \vdash e : t}{A \vdash \lambda x.e : s \to t} \tag{2}$$

$$\frac{A \vdash e_1 : s \to t \quad A \vdash e_2 : s}{A \vdash e_1 e_2 : t} \tag{3}$$

$$\frac{A \vdash e : s \quad s \leq t}{A \vdash e : t} \tag{4}$$

A partial ordering $\leq$ of the types is defined by the two rules:

$$t \leq \top \text{ , for all } t \qquad \frac{s' \leq s \quad t \leq t'}{s \to t \leq s' \to t'}$$

Prove in reasonable detail that for all closed n, we can derive $\emptyset \vdash n : \top$.