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Lecture Note 5: Flow Analysis

Contents

1 Motivation

Motivation: optimizing compilers and programming environments.

Flow information is a finite set of classes.

- Suppose the flow set for the expression E is {A,B,C}.
- Invariant: E can only evaluate to **`null`** or an instance of a class mentioned in {A,B,C}.
- We can inline a message send if the flow set for the receiver is a singleton set.
- We can use a switch statement to dispatch messages if the flow set for the receiver contains a reasonably small number of elements.
- We can discard "dead code", e.g., implementations of m in classes other than A, B, and C in the example below.
- To compute flow sets we will do flow analysis.

$$\begin{array}{rl} \mathrm{E} & .\ \mathrm{m}(\ \ldots) \\ \uparrow & \\ \{\mathrm{A,B,C}\} & \end{array}$$

Flow Analysis Algorithms

- Flow analysis by
  - *solving* constraints, with a
  - fast iterative algorithm.
- Improvements by:
  - trace graph and
  - code copying.
- Useful for both statically-typed and dynamically-typed languages.

World-Assumptions

- The *closed-world* assumption:

  All parts of the program are known at compile-time. A compile-time analysis is for the whole program. Example: flow analysis like the ones to be presented in this note.
- The *open-world* assumption:

  Some parts of the program are unknown at compile-time. A compile-time analysis is local for each fragment of the program. Example: inference of principal types.

2 Flow Constraints

- Assume that all program variable and argument names are distinct (rename if necessary).
- For a program, there will be a family of flow variables.
- Use the notation: $[\![\texttt{x}]\!]$, $[\![\texttt{f}]\!]$, $[\![\texttt{E}]\!]$ for the flow variables for the instance variable `x`, the formal argument `f`, and the expression `E`.
- Use the notation $[\![\texttt{this-C}]\!]$ for the flow variable for "this" in the class `C`.
- Generate constraints from the syntax of expressions.
- Any flow analysis must be approximative.
- Too large sets are sound and conservative, but very large sets are useless.
- How precise information can we get? How fast can we compute it?
- We will work with flow constraints of three forms:

$$
\begin{array}{rcll}
c & \in & X & \textit{start constraints} \\
X & \subseteq & Y & \textit{propagation constraints} \\
(c \in X) \Rightarrow (Y & \subseteq & Z) & \textit{conditional constraints}
\end{array}
$$

  There is always a unique minimal solution.

Flow Constraints

12) "$\textsc{id}$ `=` $\textsc{exp}$". For the assignment we have:

$$[\![\textsc{exp}]\!] \subseteq [\![\textsc{id}]\!]$$

16) "**`this`**". If **`this`** occurs in a method in class C, then we have

$$\mathtt{C} \in [\![\mathtt{this\text{-}C}]\!]$$

17) "**`new`** `C()`". For object creation we have

$$\mathtt{C} \in [\![\mathtt{new\ C()}]\!]$$

21) "$\textsc{exp}$. $\textsc{id}$ `(` $\textsc{exp}_1, \ldots, \textsc{exp}_n$ `)`". For a message send we have:

$$\mathtt{C} \in [\![\textsc{exp}]\!] \Rightarrow \begin{cases} [\![\textsc{exp}_1]\!] \subseteq [\![\textsc{id}_1]\!] \\ \vdots \\ [\![\textsc{exp}_n]\!] \subseteq [\![\textsc{id}_n]\!] \\ [\![\textsc{exp}_0]\!] \subseteq [\![\textsc{exp}\mathtt{.}\textsc{id}\mathtt{(}\textsc{exp}_1\mathtt{,\ldots,}\textsc{exp}_n\mathtt{)}]\!] \end{cases}$$

where `C` implements a method for the message:

> type $\textsc{id}$`(` type_1 $\textsc{id}_1, \ldots,$ type_n $\textsc{id}_n$`) {`
> $\textsc{exp}_0$
> `}`

Example

Suppose the classes `A`, `B`, `C`, `D` implement the interface `I`.

```
class A implements I {
  I x = new D();
  public I m(I f) {  // flow info for f = { B }
    return f.m(x);
  }
}

class B implements I {
  public I m(I g) {  // flow info for g = { C,D }
    return this;
  }
}

... new A().m(new B()).m(new C()) ...

// flow info for  new A().m(new B())  = { B }
```

Constraints for the Example

$$\texttt{D} \in [\![\texttt{x}]\!]$$

$$\texttt{B} \in [\![\texttt{this-B}]\!]$$

$$\texttt{A} \in [\![\texttt{new A()}]\!] \qquad \texttt{B} \in [\![\texttt{new B()}]\!]$$

$$\texttt{C} \in [\![\texttt{new C()}]\!] \qquad \texttt{D} \in [\![\texttt{new D()}]\!]$$

$$\texttt{A} \in [\![\texttt{f}]\!] \Rightarrow \begin{cases} [\![\texttt{x}]\!] \subseteq [\![\texttt{f}]\!] \\ [\![\texttt{f.m(x)}]\!] \subseteq [\![\texttt{f.m(x)}]\!] \end{cases}$$

$$\texttt{B} \in [\![\texttt{f}]\!] \Rightarrow \begin{cases} [\![\texttt{x}]\!] \subseteq [\![\texttt{g}]\!] \\ [\![\texttt{this-B}]\!] \subseteq [\![\texttt{f.m(x)}]\!] \end{cases}$$

$$\texttt{A} \in [\![\texttt{new A()}]\!] \Rightarrow$$
$$\begin{cases} [\![\texttt{new B()}]\!] \subseteq [\![\texttt{f}]\!] \\ [\![\texttt{f.m(x)}]\!] \subseteq [\![\texttt{new A().m(new B())}]\!] \end{cases}$$

$$\texttt{B} \in [\![\texttt{new A()}]\!] \Rightarrow$$
$$\begin{cases} [\![\texttt{new B()}]\!] \subseteq [\![\texttt{g}]\!] \\ [\![\texttt{this-B}]\!] \subseteq [\![\texttt{new A().m(new B())}]\!] \end{cases}$$

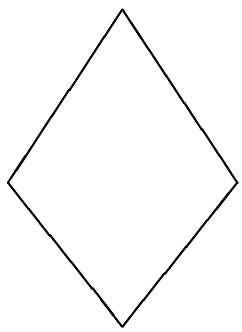
$$\texttt{A} \in [\![\texttt{new A().m(new B())}]\!] \Rightarrow$$
$$\begin{cases} [\![\texttt{new C()}]\!] \subseteq [\![\texttt{f}]\!] \\ [\![\texttt{f.m(x)}]\!] \subseteq [\![\texttt{new A().m(new B()).m(new C())}]\!] \end{cases}$$

$$\texttt{B} \in [\![\texttt{new A().m(new B())}]\!] \Rightarrow$$
$$\begin{cases} [\![\texttt{new C()}]\!] \subseteq [\![\texttt{g}]\!] \\ [\![\texttt{this-B}]\!] \subseteq [\![\texttt{new A().m(new B()).m(new C())}]\!] \end{cases}$$

The Space of Flow Sets

- Each flow variable ranges over sets of classes, i.e., we have the lattice:

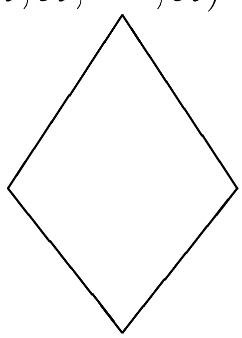
$$\mathcal{U}$$



$$\emptyset$$

- The tuple of *all* flow variables ranges over the lattice:

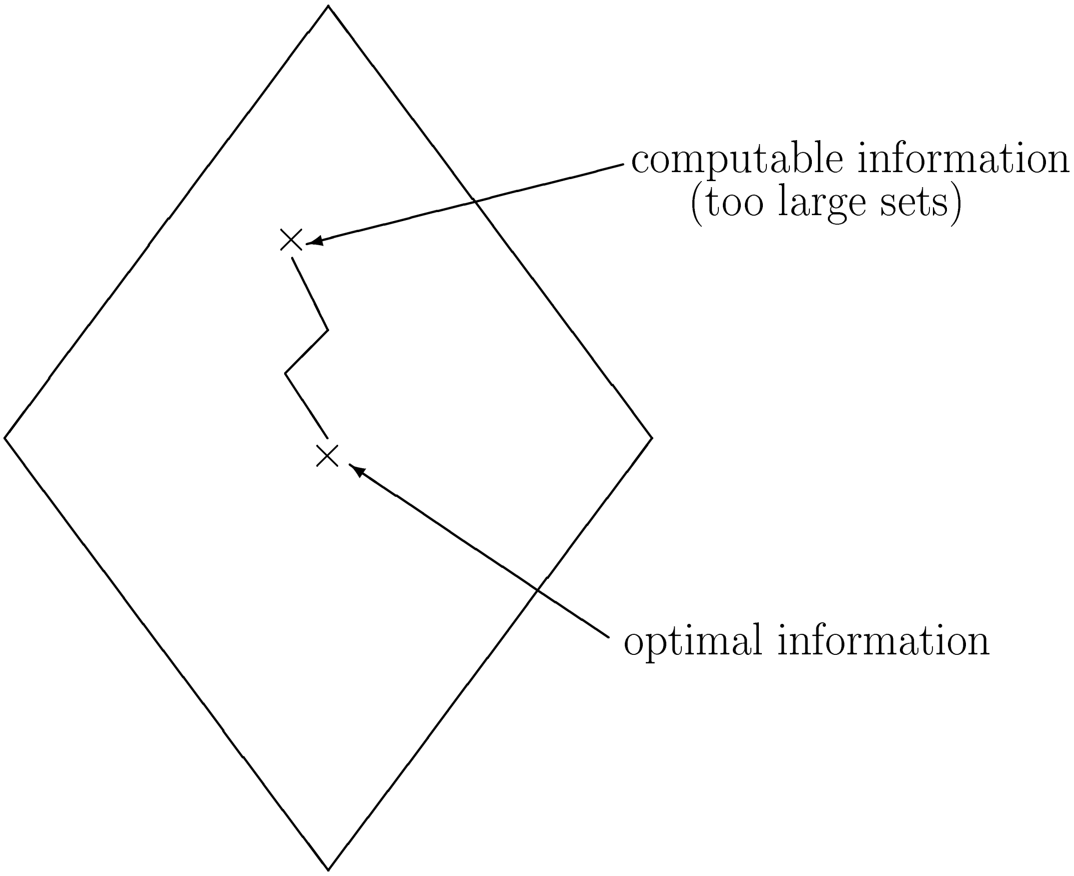
$$(\mathcal{U}, \mathcal{U}, \ldots, \mathcal{U})$$



$$(\emptyset, \emptyset, \ldots, \emptyset)$$

- Nice mathematical properties.

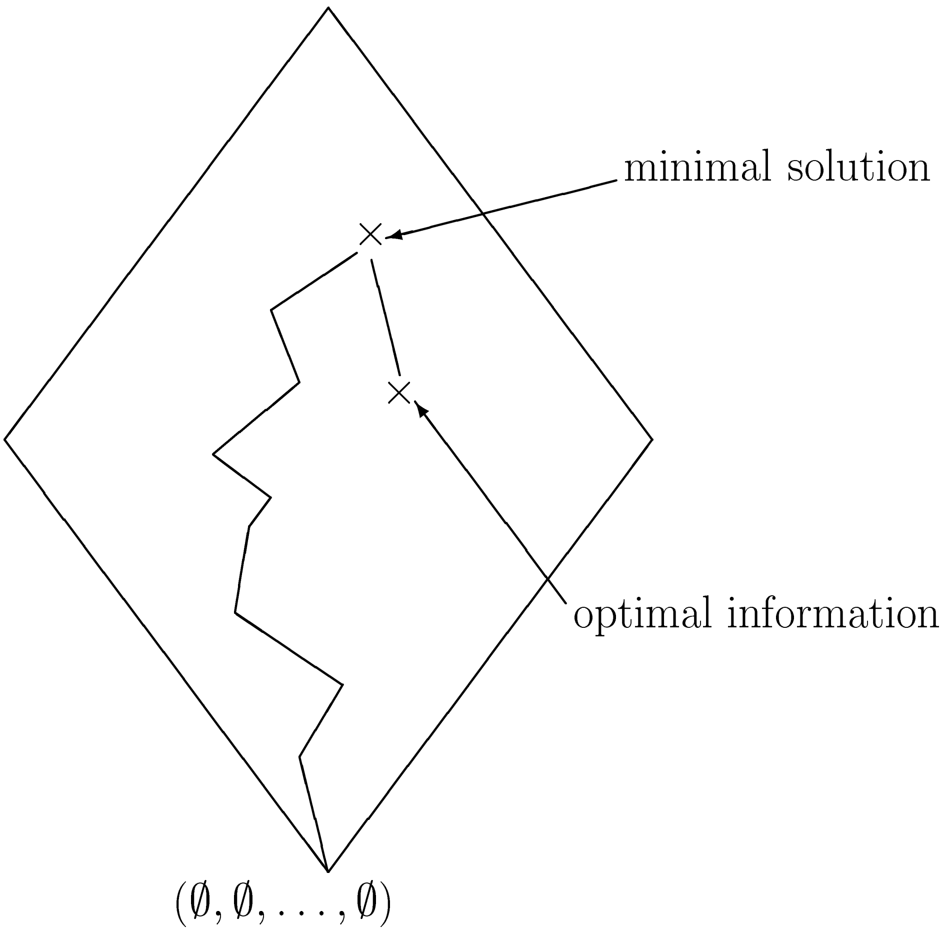
Optimal and Computable Information



- Computable information is above optimal information.
- The top of the lattice corresponds to trivial information.

Algorithm

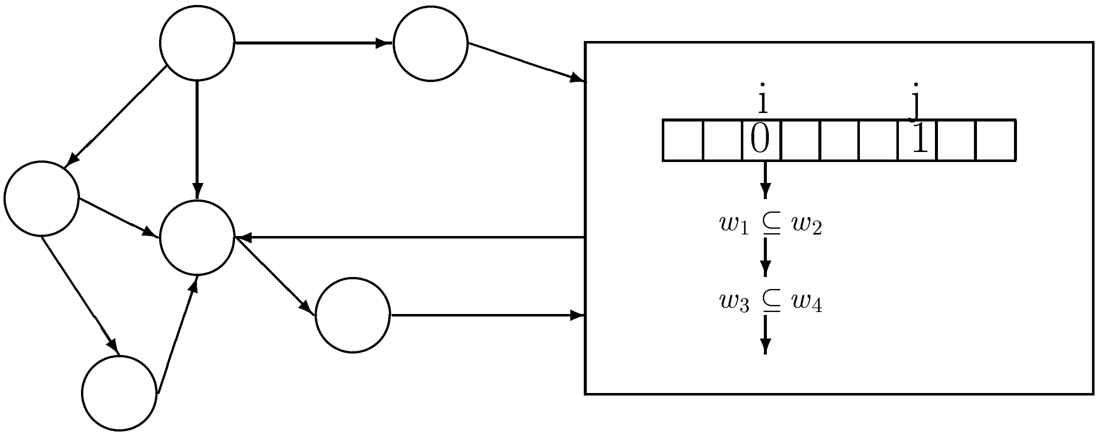
- The minimal solution can be computed in worst-case cubic time.
- Iterate towards the minimal solution:



- Use clever data structures.

3 The Constraint Solver

- Accepts constraints one at the time.
- Maintains the minimal solution.
- Its internal data structure is a graph.
- The graph has one node for each flow variable.
- An edge $v \rightarrow w$ implies $v \subseteq w$.
- The value of each flow variable is recorded in a bit vector.
- Conditional constraints are tied to the relevant bit.



- $B(X)$ = *bitvector in node X*
- $K(X, i)$ = *list of pending constraints for bit i in node X*

Solver Operations

$\textsc{insert}(C \subseteq X) =$
    for $i \in C$ **do** $\textsc{propagate}(X,i)$ **end**

$\textsc{insert}(X \subseteq Y) =$
    create an edge $X \rightarrow Y$;
    for $i \in B(X)$ **do** $\textsc{propagate}(Y,i)$ **end**

$\textsc{insert}(c \in X \Rightarrow Y \subseteq Z) =$
    if $B(X, c)$ **then**
      $\textsc{insert}(Y \subseteq Z)$
    else
      $K(X, c) = K(X, c){+}{+}(Y \subseteq Z)$
    end

$\textsc{propagate}(v,i) =$
    if $\neg B(v, i)$ **then**
      $B(v, i) =$ true;
      for $v \rightarrow w$ **do** $\textsc{propagate}(w,i)$ **end**;
      for $k \in K(v, i)$ **do** $\textsc{insert}(k)$ **end**;
      $K(v, i) = ()$
    end

An Amortized Analysis

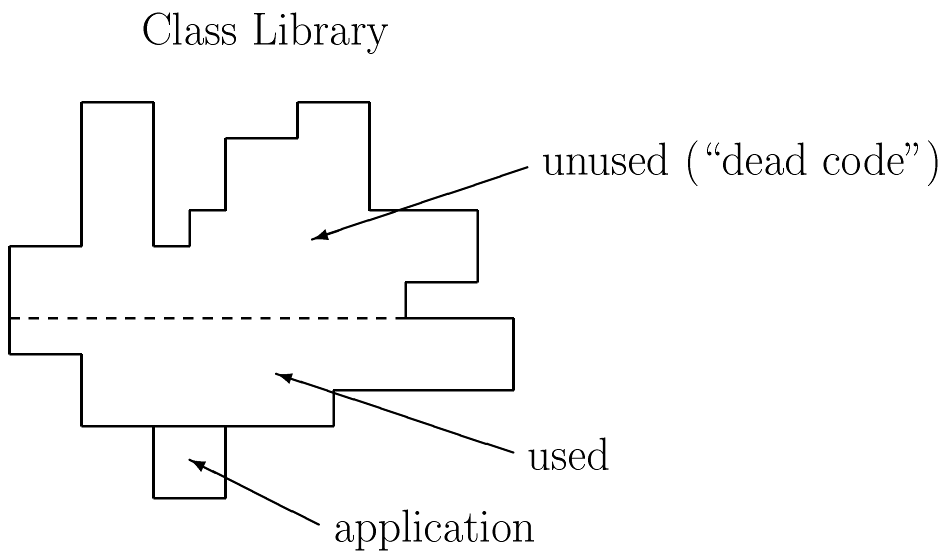
- Each activity uses up some part of the graph.
- Each bit is propagated along a specific edge at most once.
- Since there are $O(n^2)$ edges this totals $O(n^3)$.
- Each of the $O(n^2)$ constraints may:
  - be inserted into and deleted from a list once; and
  - cause the creation of a single edge,
- Which is $O(1)$ for each constraint.
- The total cost is thus $O(n^3)$.
- In practice, locality in programs gives more like linear behavior.

4 Quality of Information

- The basic algorithm is not very precise.
- It has several weak spots:
  - huge class libraries;
  - polymorphic methods; and
  - polymorphic container classes.
- They can be improved by:
  - dead code detection; and
  - code copying.
- A series of papers and PhD theses have given ever better techniques to handle these problems.
- Some techniques are general.
- Others require insight into the specific language in question.
- This note explains the idea of how to copy methods and classes in a rather static way which may square the size of the program.
- Only *adaptive* code-copying strategies seem to work in practice.

The Trace Graph

- For a given application, most of the class library counts as "dead code".
- A trace graph gives a conservative estimate of code that can be ignored.
- This gives both more precise information and a considerable speed-up of the analysis.
- The key idea: reachability.



The Extended Solver

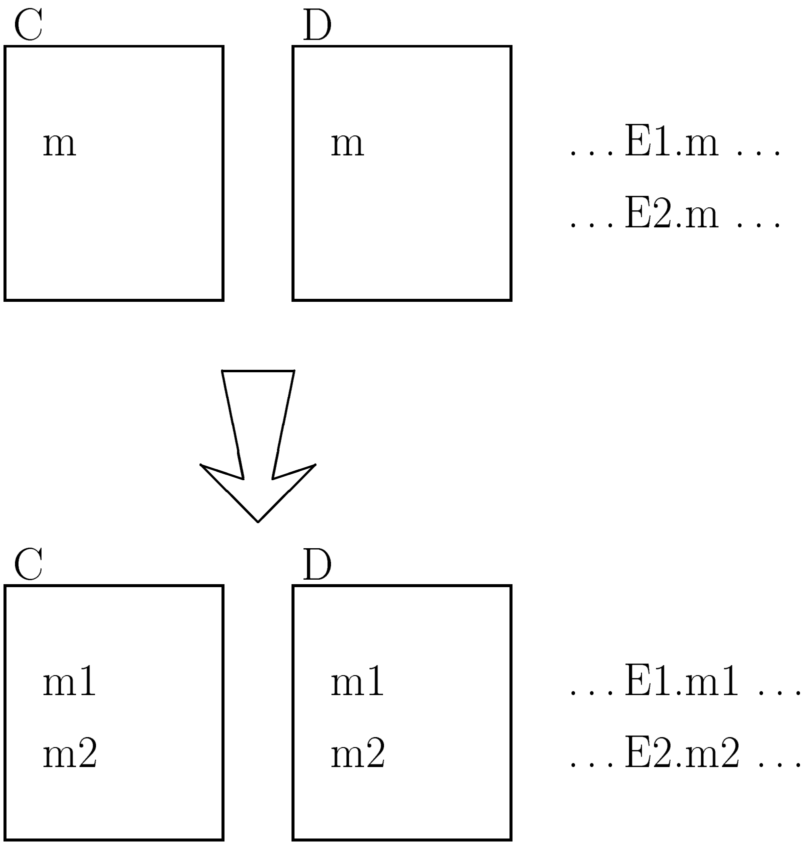
- Maintain a bit vector of *live* methods.
- Only insert constraints from the reachable parts.

> L = $\emptyset$;
> **for** $k \in$ CONSTRAINTS(main) **do**
>   INSERT(k)
> **end**;
> **while** L$\neq$LIVE **do**
>   choose m from LIVE$\setminus$L;
>   **for** $k \in$ CONSTRAINTS(m) **do**
>   INSERT(k);
>   L = L$\cup$\{m\}
>   **end**
> **end**;
> remove all methods not in L

- The algorithm is still $O(n^3)$.
- The flow sets of live methods may be improved in this way.

Copying Methods

- A program transformation to handle polymorphism.
- Creates a (nominal) copy of each method implementation for each syntactic invocation.
- Can be iterated to yield ever better information.



Example

Suppose the classes `A` and `B` implement the interface `I`.

A polymorphic use of the identity function:

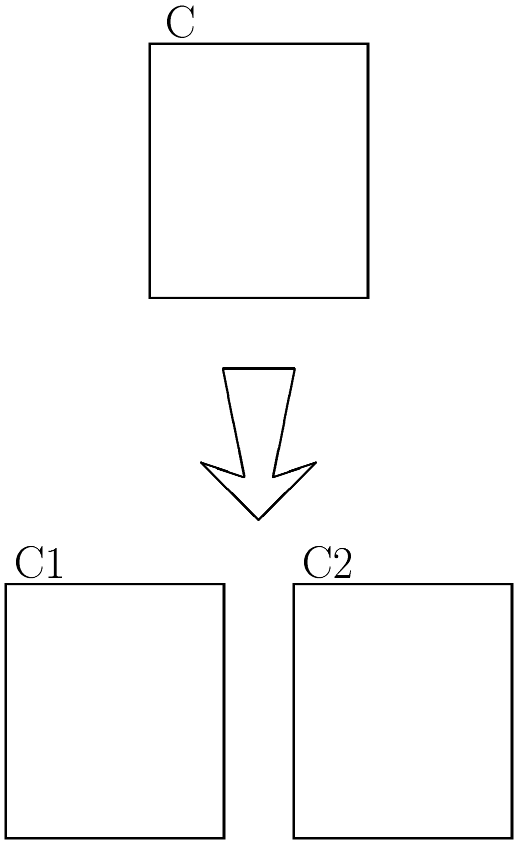
```
class C {
  I id(I x) {
    return x;  // flow set for x = { A,B }
  }
}
new C().id(new A()).m(5);
new C().id(new B()).m(5);
```

and a version where we make two copies of `id`:

```
class Cprime {
  I id1(I x) {
    return x;  // flow set for x = { A }
  }
  I id2(I x) {
    return x;  // flow set for x = { B }
  }
}
new Cprime().id1(new A()).m(5);
new Cprime().id2(new B()).m(5);
```

Copying Classes

- A program transformation for container classes that gives polymorphic instance variables.
- Creates a (nominal) copy of each class for each syntactic “**`new C()`**”.
- Can be iterated to yield ever better information.



...**`new C()`** ...

...**`new C()`** ...

...**`new C1()`** ...

...**`new C2()`** ...

Example

A polymorphic use where `A` and `B` implement `I`:

```
class Container {
  I x;  // flow set for x = { A,B }
  Container put(I val) { x = val; return this;}
  I get() { return x; }
}
new Container().put(new A()).get().m(5);
new Container().put(new B()).get().m(5);
```

and a version where we make two copies of `Container`:

```
class Container1 {
  I x;  // flow set for x = { A }
  Container1 put(I val) { x = val; return this;}
  I get() { return x; }
}
class Container2 {
  I x;  // flow set for x = { B }
  Container2 put(I val) { x = val; return this;}
  I get() { return x; }
}
new Container1().put(new A()).get().m(5);
new Container2().put(new B()).get().m(5);
```