CS 565 December 16, 1997
Jens Palsberg

# Final Exam + Qualifying Exam

Submit your solutions on separate paper (don't submit this handout). Write your name and id number on the top of the first page you submit. I will stable your pages together. This exam is intended to take you 2 hours. Budget your time wisely. There are 4 questions on the exam. Each question is worth 5 points, for a total of 20 points maximum. Write the solutions for different questions on different sheets of paper. Make clear what problem each solution is for. I have spare paper, just ask for it.

To those who are taking the qualifying exam: to pass the exam you must get an excellent score.

**1.** [**Simple Types**] Consider the following $\lambda$-term:

$$F = (\lambda f.((\lambda b.\lambda z.((bf)f)z)(\lambda x.\lambda y.xy)))(\lambda a.\lambda b.b) \ .$$

I have used the standard convention that the body of a $\lambda$-abstraction extends "as far as possible." Write down a constraint system for $F$ which captures typability with simple types. Decide if $F$ has a simple type or not by doing unification of the constraints. If $F$ has a simple type, then give a type derivation.

**2.** [**Structural Subtyping**] Here are two types $s, t$:

$$\begin{aligned} s &= \mu\alpha.((\top \to \alpha) \to (\alpha \to \bot)) \ , \\ t &= \mu\beta.((\bot \to \beta) \to (\top \to \beta)) \ . \end{aligned}$$

Decide whether $s \leq t$, in the Amadio/Cardelli type ordering. Show details of how you do it.

**3.** [**Control-Flow Analysis**] Consider the following extension of the pure $\lambda$-calculus. There are four new operators $\mathsf{triple}$, $\mathsf{first}$, $\mathsf{second}$, $\mathsf{third}$. The grammar is:

$$e \quad ::= \quad x \mid \lambda x.e \mid e_1 e_2 \mid \mathsf{triple}\ e_1 e_2 e_3 \mid \mathsf{first}\ e \mid \mathsf{second}\ e \mid \mathsf{third}\ e$$

The operator $\mathsf{triple}$ takes three arguments, evaluates them to values, and creates a tuple which contains the three values. The operator $\mathsf{first}$ extracts the first element of a tuple, the operator $\mathsf{second}$ extracts the second element of a tuple, and the operator $\mathsf{third}$ extracts the third element of a tuple. Specify a 0-CFA-style flow analysis which for a given program in this language can, for every subexpression, give a conservative estimate of which functions and/or triples can be the result of evaluating that subexpression.

**4.** [**Fast Control-Flow Analysis**] Consider the following lambda-term:

$$G = (\lambda^1 v.(((\lambda^2 x.\lambda^3 k.kv)v)(\lambda^4 a.a)))(\lambda^5 y.\lambda^6 m.my) \ .$$

Show the graph for $G$ that is used by the Heintze/McAllester $O(n^2)$ time flow analysis algorithm. What is the label set for $G$ produced by the algorithm?