

# DIFFERENTIAL ON-LINE SENSOR CALIBRATION

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## ABSTRACT

We have developed an on-line calibration scheme that employs a single source as the external stimulus that creates differential sensor readings used for calibration. The technique utilizes the maximal likelihood principle and a nonlinear system optimization solver to derive the calibration function of arbitrary complexity and accuracy. The approach is intrinsically localized and we present two variants: i) one where only two neighboring sensors have to communicate in order to conduct calibration; ii) one where a provably minimum amount of communication is achieved. We evaluate the techniques using traces from light sensors recorded by deployed sensors, and statistical evaluations are conducted in order to obtain the interval of confidence to support all the results.

## 1. INTRODUCTION

We distinguish between the time-invariant systematic bias and the random noise component of the error (Figure. 1), and focus on identifying and correcting the bias. We utilize an external stimulus in intervals when all other sources of excitation are time-invariant and the differential impact on two or more sensors is then captured. The technique is demonstrated on a set of light intensity measurements recorded by deployed sensors.

The key idea of our technique is that we use an actuator to produce differential simultaneous excitement of all sensors. The approach has a number of novel properties. First, it is maximally localized in that each sensor only needs to communicate with one other sensor in order to be calibrated. This is also the first variant of the approach. In addition, the number of time steps that are required for calibration is very low. We have developed another version of the localized algorithm that utilizes an integer linear programming (ILP) formulation to provably minimize the

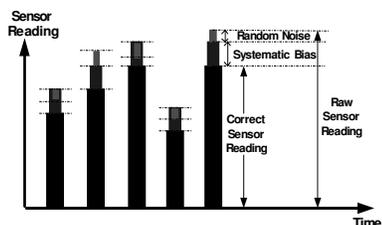


Figure 1. Systematic bias and random noise of sensor measurements.

required number of packets that must be sent for calibration (the second variant of the approach). In many senses, the second variant is an elaboration of the first variant. Conceptually, only one sensor needs to

broadcasts its own measurements to the entire network in order to calibrate the rest of sensors. The on-line calibration techniques have many benefits including suitability for robust enhancement using either consistency checking or sensor fusion. In addition to maximal likelihood, the technique can also provide other types of calibration, including one where the maximal error is minimized or the sum of weighted expected error is minimized.

## 2. RELATED WORK

In this Section, we survey the related work along the following lines: sensor data calibration, linear and nonlinear function optimization. Some of the existing state-of-the-art calibration techniques for sensor networks include [3][6][8][9][13][14]. Until now, calibration was addressed in sensor networks mainly in relationship to location discovery. The Medusa system from UCLA [13] and SpotON system from University of Washington [8] are two state-of-the-art efforts that have been reported in this domain. The emphasis in both efforts is on building models of the signal strength and the distance for a specific set of radio transceivers and receivers in off-line calibration.

The localized calibration approach proposed by Bychkovskiy et al. [3] was based on the assumption that physically closed sensors usually have temporarily correlated readings. They first consider pairs of physically close sensors and then try to find the most consistent way to simultaneously satisfy all pair-wise relationships. More recently, Ihler et al. [9] reformulate the self-calibration problem within a graphical model framework based on the observation that the information used for sensor calibration is fundamentally local with regard to the network topology. Then the nonparametric belief propagation (NBP) is applied for both estimating sensor locations and representing location uncertainties. Calibration can also take other forms such as time synchronization [6].

Linear programming and nonlinear programming have been the popular optimization mechanisms since the 1940's. Both of these methods involve three entities: variables, an objective function, and constraints. The objective is to find a set of assignments to the variables in such a way that the objective function is minimized or maximized, and at the same time have all the constraints remain satisfied. Standard references for linear programming include [10][11][12]. For nonlinear programming, some useful sources include [1][2][10][11][12]. In our research, we chose to use the unconstrained nonlinear function

minimization techniques. More specifically, we used the public available software package WNLIB [15]. The nonlinear programming problem instance was solved by weighting all constraints into the objective function and relying on the conjugate direction-based unconstrained nonlinear function minimization techniques provided by WNLIB [16].

### 3. ACTUATOR-BASED CALIBRATION

Our actuator-based calibration is intrinsically amenable to local execution and we discuss two approaches to conduct such on-line calibration while keeping the communication cost low. Although we restrict our discussion to the light sensors example for demonstration purpose, the approach is generic and can be easily retargeted to other types of sensors. We start with the description of all the underlining abstractions and assumptions and the novelty of our approach at the conceptual level. After that, we present the problem formulation and discuss the optimization conditions.

#### 3.1 Assumptions

There are two assumptions that our approach is based upon. The first one assumes the stability of the stimuli and the environment the sensors are deployed in. More specifically, we assume that there exist periods of time moments/snapshots where the sensor readings are relatively stable in order to conduct calibration. The stableness is measured by considering two consecutive time moments and comparing the similarity between the two sensor readings given a certain stableness tolerance bound. Figure 2 shows the probabilities of percentage of stable sensors given various similarity tolerance bounds. Figure 3 shows the length of the stableness for six randomly selected sensors. The length of the stableness is measured in terms of the number of consecutive stable sensor readings. In Section 3.2, we show that this assumption is indeed reasonable and can be satisfied in order to conduct calibration

The second assumption is the independence of measurement errors. However, this assumption only holds when the maximal likelihood principle is applied where the probabilities of errors are multiplied. If other traditional objectives such as  $L_1$  or  $L_2$  norms are used, this assumption is no long necessary.

#### 3.2 Problem Formulation

Consider a sensor network with  $M$  light sensors deployed in a field, either open or closed, where there exist light sources, reflection surfaces, and blocking items. We assume that

1. Each sensor knows its location and orientation. In our experiments, we assume this information is accurate and error-free. However, it is easy to estimate the impact of location and orientation errors on the effectiveness of our procedure experimentally using simulation.
2. A single point light source is available and is placed in a way that all sensors will receive some additional light from this source, either directly or through the reflection surfaces.

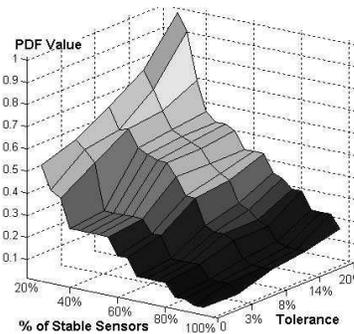


Figure 2. The measured vs. the correct distances.

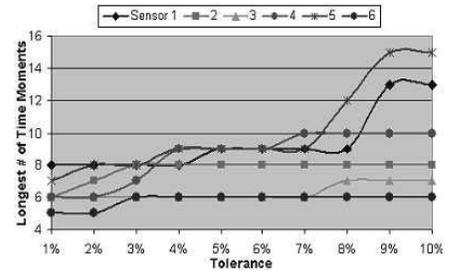


Figure 3. The measured vs. the correct distances.

3. The noise model and calibration function model are available from, for example, the proposed off-line calibration or other alternative approaches.
4. There exists a constant linear factor, denoted by  $C_i$  for sensor node  $i$ , that captures the impact of environment on each sensor's readings of the introduced point light.

Suppose that each sensor takes the measurement at  $T$  distinct time moments. These moments are selected in such a way that there is no or very small influence from other light sources and the environment. In other words, if the sensors have exactly the same readings before and after we apply the additional light source, we will be able to conclude that there is a high likelihood that the non-controllable light sources and the environment are time-stationary.

Let  $I_1, I_2, \dots, I_T$  be the intensity of the light source at such moments, which are unknown. Let  $\{r_{i1}, r_{i2}, \dots, r_{iT}\}$  and  $\{r_{i1}'' , r_{i2}'' , \dots, r_{iT}''\}$  be the light intensity observed by sensor  $i$  before and after the introduction of the additional light source  $I$ . Therefore, the contribution of the additional light source  $I$  at sensor  $i$  at moment  $j$  is  $r_{ij} = r_{ij}'' - r_{ij}$ . Let  $B_i$  be the bias that all measurements of sensor  $i$  should be modified. Our goal is to determine  $B_i$  such that bias-corrected sensor reading  $(r_{ij} + B_i)$  is as accurate as possible. Moreover, we want to determine  $B_i$  with information from the fewest possible sensors and time moments. Meanwhile, we also have the following optimization objectives: (1) Keep the communication cost and therefore energy consumption low. This is achieved by minimizing the number of messages exchanged among sensors for the purpose of calibration. (2) Obtain calibration with high quality in terms of the accuracy of the calibration bias  $B_i$ . (3) Ensure the reliability of the calibrated results. This is accomplished by providing the interval of confidence for each calibration value of each sensor.

It is important to note that although we measure both the environmental impact coefficient and the bias value linearly in the above discussion, it is easy to generalize our technique to the case when one or both of them follow an arbitrary piece-wise polynomial function. In that case, a calibration function of arbitrary form can be incorporated as long as it can be computed.

At the conceptual level, our approach consists of five attributes. 1) To the best of our knowledge, this is the first technique that uses additional actuators (source of light) to enable on-line in-field calibration in such a heterogeneous and complex environment. 2) We use noise model, calibration function model, and the environmental impact model with respect to the light source in order to conduct maximal likelihood-based calibration.

3) We apply nonlinear function minimization solver to solve the problem and obtain results. In principle, this enables us to treat arbitrary forms of noise, calibration function and environmental impact dependencies. 4) We use the standard boosting re-sampling technique to obtain the interval of confidence for each sensor as well as for all sensors overall. 5) Our approach is amenable to analytical studies as we can derive conditions in terms of required time snapshots and sensors in order to get meaningful calibration values.

In the ideal case when there is no bias, the differential reading  $r_{ij}$  at sensor  $i$  is equal to the product of the unknown intensity of the source  $I_j$  and the environmental impact constant  $C_i$ . (i.e.  $r_{ij} = I_j \cdot C_i$ ). However,  $r_{ij}$  is not bias-free in real life and thus needs to be corrected by the bias value  $B_i$ . Therefore for each sensor  $i$  at moment  $j$ , we have

$$r_{ij} + B_i = I_j \cdot C_i \quad (1)$$

We can rewrite this equation, when considering the existence of the random noise  $\varepsilon_{ij}$ , as follows:

$$(r_{ij} + B_i) - I_j \cdot C_i = \varepsilon_{ij} \quad (2)$$

Our first solution is to use the gradient direction-based nonlinear function solver (the term  $I_j \cdot C_i$  makes this system nonlinear) to determine the  $B_i$ ,  $I_j$ , and  $C_i$  by optimizing a selected objective function based on the combination of each individual error  $\varepsilon_{ij}$ . For example, we can minimize the total of these discrepancies in  $L_1$  or  $L_2$  norm. Another alternative, which is more attractive both conceptually and practically, is to use the maximal likelihood principle to maximize the probability of the selected calibration values. More specifically, we want to find the bias correction values in such a way that the probabilities of the corresponding errors produced from a particular instance are maximized.

Let  $M$  be the error model constructed in off-line calibration [7] and  $P_{ij} = M(\varepsilon_{ij})$  be the probability that a particular error  $\varepsilon_{ij}$  is detected. Note that the  $\varepsilon_{ij}$ 's are the exact noise (well captured from our off-line error model for example) in our environment. Assuming that the errors are independent, we can now formulate our calibration problem as an instance of maximal likelihood-based nonlinear function minimization with minimizing

$\prod_{ij} P_{ij}$  as the objective function. In the actual implementation, we take the logarithm of each probability and maximize the summation of logarithms.

It is interesting and instructive to analyze the number of measurements (i.e. sensors) and the time snapshots required to obtain properly constrained and over-constrained nonlinear systems. Consider  $M$  sensors,  $T$  time snapshots, and given that the number of parameters of the bias correction function  $B_i$  is  $U$ , and the environmental impact function  $C_i$  has number of parameters  $V$ . The total number of equations that can be constructed is  $M \cdot T$ , and the total number of unknown variables is  $(U+V)M+T$ . In a linear system, there must be an equal or greater number of equations than the number of variables in order for the system to have a unique solution. Similarly, in the nonlinear system, we constrain the number of unknown variables to be less than or equal to the number of equations (i.e.

$M \cdot T \geq (U+V)M+T$ ). Table 1(a) provides a summary of the number of needed snapshots given the number of sensors when fixing  $U=2$  and  $V=2$ ; Table 1(b) shows the required number of sensors given the limited number of snapshots. Note that “-” indicates that the given condition is unsolvable.

Once the solution is available, we establish the interval of confidence for each variable or an overall consistency of the system using the resubstitution method [4][5]. We randomly select  $k\%$  of the equations (e.g. 70% in our experiments). For this subset of equations, the system of nonlinear equations is transformed to a nonlinear function minimization instance and solved using the conjugate direction-based solver. We record various values of the variables of interest. This procedure is then repeated a large number of times, specifically 200 in our experiments in order to construct the interval of confidence.

# OF SENSORS	# OF SNAPSHOTS
1	-
2	8
3	6
4	6
5	5
6	5
7	5
8	5

# OF SNAPSHOTS	# OF SENSORS
1	-
2	-
3	-
4	-
5	5
6	3
7	3
8	2

Table 1(a). necessary # of time moments.

Table 1(b). necessary # of sensors.

## 4. EXPERIMENTAL RESULTS

The main goal for the experiments is to demonstrate the two variants of our localized on-line calibration techniques under various scenarios. The calibration error is defined by normalizing the difference between the calibrated value and the measured value against the difference between the correct and the measured values. Traces from actual deployed light sensors were used to evaluate the two variants: 1) *only two neighboring sensors have to communicate in order to conduct calibration*; 2) *a provably minimum amount of communication is achieved*. The interval of confidence is then computed for all obtained results.

In our experimental setup, eight stationary light sensors are positioned on a flat surface; a light bulb held above the sensors is used as the external stimulus. Each light sensor consists of a miniature silicon solar cell that records the light intensity readings and converts light impulses directly into electrical charges (photovoltaic). The silicon solar cell generates its own power and does not require any external bias unlike other conventional photo diodes or transistors. Furthermore, this silicon cell is mounted on a 0.78cm x 0.58cm x 0.18cm thick plastic carrier and generates about 400mV in moderate light,

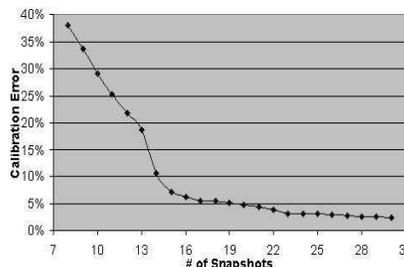


Figure 4. Performance of the 1<sup>st</sup> variant of the on-line calibration.

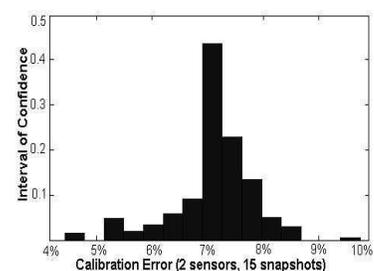


Figure 5. Interval of confidence of the calibration error.

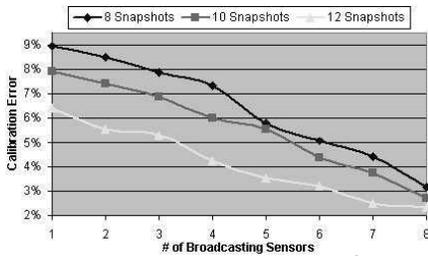


Figure 6. Performance of the 2<sup>nd</sup> variant of the on-line calibration.

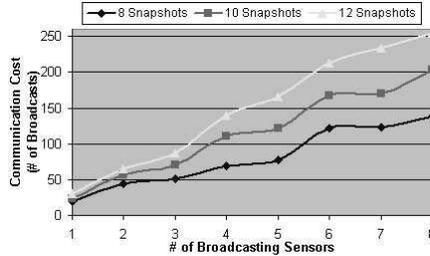


Figure 7. Communication cost of the 2<sup>nd</sup> variant of the on-line calibration.

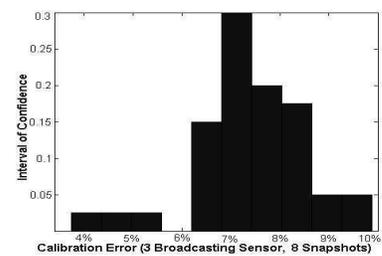


Figure 8. Interval of confidence of the calibration error.

which is similar to a typical room. Detailed description of the light appliances can be found in [16].

Figure 4 gives the performance in terms of calibration error for the first variant of the localized on-line calibration technique. We randomly generated two sensors of arbitrary locations and an additional light source that contributes the differential light intensity. The sensor intensity measurements are obtained by first calculating the correct sensor intensities following linear environmental impact models (i.e.  $V=2$ ); then we modify the correct sensor intensities by a linear error model (i.e.  $U=2$ ). Finally, on top of this value, we introduce extra random noise that follows the Gaussian distribution. As we can observe from the figure, once the number of snapshots is above 16 (the required number of snapshots is eight given two sensors), the calibration error improves little even when extra snapshots are introduced. Finally, Figure 5 gives the interval of confidence under the conditions of two sensors, 15 snapshots, and both the calibration model and the environmental impact model take linear form (i.e.  $U=V=2$ ). The following conclusion can be drawn from the figure: with 92% of confidence, the calibration error is bounded by  $7.3\% \pm 0.5\%$ .

For the second variant of the calibration approach, our main goal is to study how the number of broadcasting sensors impacts the calibration error and the communication cost (note that it is sufficient for only one sensor to broadcast in order to conduct calibration for all eight sensors). Figure 6 shows the calibration error when the number of broadcasting sensors varies from 1 to 8. We exhausted all possible combinations of broadcasting sensors under three different numbers of snapshots and the condition  $U=V=1$ . The results shown in figure are the average. Figure 7 gives the average communication cost measured in terms of the number of broadcasting packets sent under the same conditions. Finally, the confidence interval of the calibration error in eight snapshots when three sensors broadcast is shown in Figure 8. As the figure indicates, with 82% of confidence, the calibration error is bounded by  $7.5\% \pm 0.5\%$ .

## 5. SUMMARY

We have developed a time-variant actuator-based approach to address the sensor calibration problem. We first formulate the problem as a nonlinear function minimization instance. Then we combine the maximal likelihood principle and numerical optimization techniques to obtain solutions. This approach is generic in that arbitrary forms of the calibration model, error model and the environmental impact function can be adopted. In addition, it can be easily modified to retarget other types of measurements. Comprehensive experiments on deployed sensor

measurements are conducted; accuracy bounds expressed in terms of confidence intervals support all the results.

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