

1 Ways to Number

There are ten fingers on two hands. But counting the spaces between the fingers gives a total of eight. When counting on our fingers the tenth starts again. But some in the Amazon basin count on the spaces in between fingers. That would make the eighth place the *start-again* location.

A lot of things aren't grouped *ten-at-a-time*. *Dozen*, a word like *ten*, groups eggs. *A minute* groups sixty seconds. *A week* groups seven days.

Grouping by a special number (ten, twelve, sixty, seven) defines a *base*. A base is always one more than the largest value shown by a single digit. It is also always the number of single digit numbers, where we always begin with zero for none or nothing. A number n written when the base is b could use digits drawn from $0, 1, \dots, b - 1$. Suppose the string representing n to base b is called x . That could be written:

$$n_{10} = x_b \tag{1}$$

Computers and other digital systems use a different base than ten. These three are most useful:

Binary or base 2 – numbers written using just 0, 1

Example:

$$10_2 = 2_{10} \tag{2}$$

Octal or base 8 – numbers written using just

0, 1, 2, 3, 4, 5, 6, 7;

Example:

$$543_8 = 320 + 32 + 3 = 355_{10} \tag{3}$$

Hexadecimal or base 16 – numbers written using just

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, *X, L, C, D, E, F* or 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, *A, B, C, D, E, F*

Example:

$$27D_{12} = 2 \times 256 + 7 \times 16 + 13 = 637_{10} \tag{4}$$

The next question tests one's understanding of bases.

The square of 24 in base b equals 554 in base b . What is base b ? [1]

Sample Solution: The statement is equivalent to this relationship:

$$(24_b)(24_b) = 554_b \tag{5}$$

That implies:

$$(4_b)(4_b) - b = 4_b \quad (6)$$

A reasonable guess is $b = 12$.

Writing each of the three expressions in the first equation in base 10, using $b = 12$ we have:

$$(2 \times 12 + 4)(2 \times 12 + 4) = 5 \times 144 + 5 \times 12 + 4 \quad (7)$$

Multiplying the four terms on the left hand side in base ten yields

$$4 \times 144 + (8 + 8) \times 12 + 16 = 576 + 192 + 16 = 784 \quad (8)$$

Evaluating the right hand side yields $720 + 60 + 4$ for the same value, 784.

References

- [1] Don Christiansen. Problem. *IEEE-USA TODAY'S ENGINEER*, page 4, 2006.