# THE EVOLUTION OF NUMBER

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## Contents

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Anthropological studies of primitive tribes have not yet disclosed a society where men lack an understanding of number. Indeed, studies by naturalists have indicated that many birds and insects ~~posses~~ possess what has been referred to as a "number sense." 1/ "If ... a bird's nest contains four eggs, one may be safely taken; but if two are removed, the bird becomes aware of the fact and generally deserts." 2/ Similarly, studies of pollinating insects 3/ indicate that they are able to recognize number in the patterns of petals on flowers. These insects have been shown able to discern whether a flower has 1, 2,3, 4, 5, 6, 8, 10 or 12 petals. This was proven by removing rays from Mexican marigolds "... so that the number [of rays] was reduced from 10 to 5, from 5 to 2, and so on. Marked bees either avoided the mutilated heads or were confused and visited both normal and abnormal flowers." 4/ These instances, and many others, indicate the existence of a "sense" related to number or symmetry in many forms of life. It seems almost certain that at some stage in his development man must have been restricted to a simple number sense in his comprehension of quantity.

Man's need for a better understanding of the numerical properties of objects probably arose out of his daily occupations. The hunter who met a friend and was unable to boast of the number of animals he had killed was in a sorry predicament, judged by modern standards. The first means of representing numbers that men discovered was use of the fingers. The matching principle involved in indicating a finger for each object is called one-to-one correspondence. The importance of man's ten fingers in numeration is indicated by the fact that "... those savages *who have not reached the stage of finger counting* are almost completely deprived of all perception of number." [5] The cardinal concept implied in matching was the basis of the first attempts to keep a record of number. Pebbles or sticks were cast into a pile, notches cut in wood or knots made in cord. The early origin of written number records is shown by pieces of carved bone which were used by cave men to record the number of animals killed. A notch was made on the bone that had the head of the particular animal carved on it. [6] The etymologies of some of the words we use to describe mathematical processes indicate how recently our ancestors made use of these simple methods of recording number. *Tally* comes from the French *tailler*, to cut, which was derived from the Latin *talea*, a stick or cutting. [7] Similarly, *calculate* and *calculus* are both derived from the Latin word for pebble - *calculus*. [8]

Representation of quantity by the principle of one-to-one correspondence was undoubtedly accompanied, and perhaps preceded, by the creation of number words. Number words can be divided into two main categories: those that arose before the concept of number unrelated to concrete objects, and those that arose after it. An extreme instance of the development of number words before the abstract concept of number is that of the Tsimshian language of a tribe in British Columbia. This language has seven distinct classes of number words: for flat objects and animals, round objects and time, long objects and trees, men, canoes, measurements and, as a recent development, for counting when no definite object is referred to.[9] In a like manner, the English language has many words for types of collections (set, flock, herd, lot, bunch, etc.) but the more general collection and aggregate are of foreign origin.[10] Another peculiarity of English, the many words for two things, led Bertrand Russell to comment, "It must have required many ages to discover that a brace of pheasants and a couple of days were both instances of the number two."[11]

The number word that is developed as an abstraction of the general quality of quantity frequently is the name of a particular collection of things whose number is the same as that which is to be expressed.[12] An indication of this is the fact that the number word "five" had the original meaning hand in many languages: pantcha was five in Sanskrit - modern Persian uses pentcha for hand; five in Russian is piat - piast is Russian for the outstretched hand.[13] However, many societies found it difficult to develop number words besides "one", "two", and "many".[14] That this is a fairly common starting point is illustrated by the dual meanings of the English thrice and the Latin ter; both mean three times or many. The process may be carried back by noting the common root of the Latin tres (three) and trans (beyond).[15]

The idea of number as opposed to the number of a specific set of things was not completely comprehended in early societies even when there was but a single word to express a particular quantity. The collection word for five brought forth the image of a hand (or another group of five objects). This image was compared with the set of things whose number was to be determined.[16] The difficulty in grasping the idea of the abstract nature of number may lie in its being a logical, rather than numerical, concept. Some experiments with children performed by Jean Piaget seem to indicate this. When a child is asked to lay out a number of blue chips equal to the number of red chips that are in a row spaced at uniform intervals his reactions will depend on his age. If the child is five or younger he will lay out a row of chips of the same length as the model row but will disregard spacing (placing his chips very close to each other) indicating that he believes that the number of chips is the same if the lengths of the rows are equal. At the age of six the child will match his chips to those on the table, but if the length of one row is increased by changing the spacing of the chips he believes that row has more chips than the other. A third stage is reached by the time the child is six and a half to seven: he realizes that the number of chips is independent of their spacing. "In short, children must grasp the principle of conservation of quantity before they can develop the concept of number." 17/

Numeration would be a cumbersome process if men found it necessary to form model collections and special words for every number. Fortunately, men discovered the principle of forming large numbers by combination of smaller numbers. The number which is the last simple number is known as the base or radix of its number system. Our system is a decimal one because all numbers after ten are compounded of the first ten numerals (eleven and twelve are compound numbers whose forms ~~is~~ are unrecognizable because they ~~x~~are Anglo-Saxon in origin (18). Among the bases used by man are two, three, four, five, six, eight, ten, twelve, twenty, and sixty. ~~Xx~~ Of these the most common are five, ten and twenty. 19 The mechanism involved in a decimal base is indicated by the literal meaning of eleven in a primitive language: a man and one on the ~~h~~ hand of another man. Of the non-decimal number systems, a typical binary system is that of a ~~xxxxx~~ tribe of the Torres Straits:

| | | |
|---|---|---|
| (1) <u>urapun</u> | (3) <u>okosa-urapun</u> | (5) <u>okosa-okosa-urapun</u> |
| (2) <u>okosa</u> | (4) <u>okosa-okosa</u> | (6) <u>okosa-okosa-okosa</u> 20 |

~~XXX~~ A typical quinary system is that of the Api language of the New Hebrides:

| | | |
|---|---|---|
| (1) <u>tai</u> | (6) <u>o tai</u> | (other one) |
| (2) <u>lua</u> | (7) <u>o lua</u> | (other two) |
| (3) <u>tolu</u> | (8) <u>o tolu</u> | (other three) |
| (4) <u>vari</u> | (9) <u>o vari</u> | (other four) |
| (5) <u>luna</u> (hand) | (10) <u>luna luna</u> | (two hands) 21 |

Quinary, decimal and vigesimal number systems arose from finger counting, the "whole man" being regarded, respectively, as a hand, both hands, or ~~and~~ the hands and feet. Binary systems are probably based on the symmetry of man's body; a Brazilian tribe developed a tertiary number system because they counted on the joints of the fingers; 22 The quaternary system arose in California because of the religious significance of the four quarters of the sky. 23 A system using eight basic numbers arose because counting was performed on the spaces between the fingers rather than on the fingers themselves. Apparently the simplest system, the binary, was adopted initially and discarded in favor of a system of higher radix in many parts of the world. The need for a system using a higher base arises from the difficulty of expressing large numbers in the binary system. The decimal and quinary systems have developed and been rapidly adopted in regions that previously used simpler systems of numeration. The existence of remnants of other number bases in our decimal system is readily apparent: having sixty minutes in an hour, <u>dozen</u> and <u>gross</u>, and the Biblical "three score and ten" are among the many examples.

We have seen that the radix of a number system arises from physiological or religious considerations. Examination of the number base from a mathematical standpoint yields several criteria for a satisfactory radix: it should be sufficiently large to express large numbers concisely, but small enough to reduce the number of numeral words to be memorized to a reasonable amount. The next quality is how factorable the base should be. Mathematicians are divided on this point. Some favor a prime number so as to eliminate the ambiguity that arises in the expression of fractions (3/25 represents itself, 6/50, 12/100, etc.). This type of radix makes all decimals non-terminating (<u>e. g.</u> 1/3 = .3333... ) and is impractical from the standpoint of everyday use. Other mathematicians favor a base that is evenly divisible by many numbers. The radix fulfilling the other requirements and that of factorability best is twelve.[24] A duodecimal system would employ two additional symbols (<u>e. g.</u> <u>X</u> for ten and <u>L</u> for eleven) and would represent 12 by <u>10</u>, 144 by

100, etc. The many factors of twelve simplify calculations by making quotients "come out even" more frequently and facilitate mental calculations: if a number ends in 0 in the decimal system we know it is divisible by 2 and 5; if it ends in 0 in the duodecimal system we know that it is divisible by 2, 3, 4, and 6. At any rate, ten would not be used as the mathematically chosen radix since it is neither prime nor highly factorable.

The universal need for describing quantity led to representation by one-to-one correspondence, development of number words, and a method for limiting the amount of number words needed to express large figures. An important contribution to numeration was the development of a group of written symbols for numbers:

> Written numeration is probably as old as private property. There is little doubt that it originated in man's desire to keep a record of his flocks and other goods. ... Archeological researches trace such records to times immemorial, and they are found in the caves of prehistoric man in Europe, Africa, and Asia. Numeration is at least as old as written language, and there is evidence that it preceded it. Perhaps, even, the recording of numbers suggested the recording of sounds. [25]

The first written numbers that were not tally marks alone probably occurred in Egypt about 3,400 B. C. [26] The Egyptian numerals employed the tally principle, as did most early systems, and were mainly straight lines.

The distinction between simple tallying and the Egyptian numeration is made because the Egyptians used special symbols to signify large quantities. The Egyptians had basic symbols for one, ten, one hundred, and other powers of ten. The value of a number was the sum of the values of the symbols comprising it, the value of a symbol being repeated the number of times that the symbol was repeated. Another system of numeration was the cuneiform system developed in Mesopotamia about 3,000 B. C. [27] The system used by the Babylonians was both sexagesimal and decimal. Ordinary computation was carried on in the decimal system and astronomical work in the sexagesimal. The Babylonians used the wedge-like imprint that the stylus made in the clay as one, sixty, three hundred sixty, and, in general, sixty raised to any power, the meaning of the symbol being derived from the context. Similarly, the symbol for ten served as ten times sixty raised to any power. As a later development, the Babylonians attempted to eliminate the indefinite representation that their system produced. "In lists where we make the entry '0', as an alternative to leaving blank, the Babylonians in some cases make use of the sign <u>ul</u>, meaning 'not', 'nothing.'" [28] The Babylonian "zero" was a punctuation mark like our dash. About 400 B. C. the Babylonian mathematician Naburianu realized the significance of <u>ul</u> and used it as we use our zero. This development was lost to the world because it was in the sexagesimal system which was incomprehensible to the neighboring peoples who used decimal systems. [29]

Fundamentally different from the cardinal numeration we have considered, were the systems of the Greeks and Hebrews. Bpth systems were derived from the Phoenician and followed an ordinal scheme of written numbers. Each of these civilizations used their alphabet to represent numbers. The Greeks originally represented numerals by the initial letter of the number word, but adopted the Phoenician convention of using the letters of the ~~x~~ alphabet in their natural succession to represent numbers. The first nine letters of the Greek alphabet were, successively, symbols for one through nine, ~~x~~ the next nine represented ten and its multiples through ninety, and the last nine, one hundred and its multiples through nine hundred (the Greeks added three letters to their alphabet in order to have twenty-seven symbols).

The Roman numeral system was a return to the cardinal principal, and was ~~xxxx~~ similar to the Egyptian. Two interesting theories have been proposed to explain the derivation of the Roman symbols for five and ten. The first considers the symbol for five as basic and regards the symbol for ten as two fives. This theory is based on the fact that the Roman "V" is heavier on the left side. Consequently, the five is regarded as a hand - the left side of the numeral being the four fingers and the right side the thumb. The second theory is based on the ~~xrixi~~ tallying origin of cardinal systems of numeration: "X" is considered an abbreviation for "IIIIIIIIII," and "V", one half of "X". [30]

While these number systems were developed in the Old World, the Mayan Indians of the Yucatan peninsula developed a vigesimal system which employed place notation. The Mayans wrote numbers in vertical columns, using only three symbols: . (one), _ (five) and (zero). Symbols on the lowest level were units, the next higher level, multiples of twenty, and successively, multiples of 360 and 7,200. As examples:


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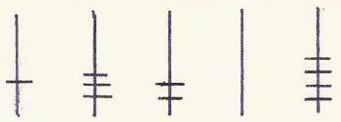
It is interesting to note the pebble and stick form that the Mayan number symbols take. The use of "stick" or straight line numerals was extremely common. The Egyptian and Roman symbols were vertical lines but horizontal forms were used in the Far East. Our "2" and "3" are derived from ~~xxxxxxxxx~~ the cursive forms of the corresponding horizontal line symbols, "z" and "ɜ".

Each of the systems of written numbers that we have discussed differ from our number system in one major respect: they were used to record figures but were not used ~~for~~ directly in calculations. A primitive calculating instrument, the abacus 32, was used, in one form or another, in almost every civilization. The abacus was a method of representing numbers in a positional notation. Reduced to its basic elements, the abacus consisted of a set of parallel columns which successively represented units, tens, hundreds, etc. Counters were placed on the columns (or marks made on columns in the dust) to indicate how many units, tens or hundreds there were in a number. The number 13,204 would be represented, by an abacus, in this manner: 

The abacus was used in calculations by adding figures to their appropriate column and transferring when the marks in a column exceeded nine. Addition and subtraction as performed on the abacus can be easily visualized, but the processes of multiplication and division are too involved to be discussed here. ~~The~~ A basic method for multiplication, "duplation", was used in Egypt at an early period (c. 3,400 B. C.), and was extremely important in Europe, surviving until the Renaissance period. Using our notation, multiplication of 32 by 17 using duplation was accomplished in this manner:

| | | |
|---|---|---|
| 1 x 32 = 32 | | |
| 2 x 32 = 64 | | |
| 4 x 32 = 128 | (4 x 32 = 2 x 64) | |
| 8 x 32 = 256 | (8 x 32 = 2 x 128) | |
| 2 x 32 = 64 | | |
| 17 x 32 = 544 | | |

Division was accomplished by the analagous "mediation". These processes were taught in universities, one institution excelling in their course in mediation, another in duplation. The product of thousands of years of civilization was an inflexible number system "so crude as to make progress well-nigh impossible" and "a calculating device so limited in scope that even elementary calculations called for the services of an expert."[33]

In India the use of a cardinal system of numbers made the abacus a fundamental tool. It was, however, difficult to re-

cord counting-board operations: the figure ≡ = could represent 32, 302, 320, and so on. In order to make an unambiguous record of a series of calculations it became customary to place a mark called <u>sunya</u> (meaning empty or blank but having no connotations of void or nothingness) to indicate an empty place on the abacus. The symbol used was O, o, or • and was eventually accepted as a number itself. It was placed at the bottom of the ordinal series 1, 2, 3 ... and was adopted by the Arabs along with the rest of the Indian numerals. The Arabs, of course, introduced "Arabic" numerals to Europe.
The Hindu-Arabic numerals greatly simplified all calculations; their effectiveness is illustrated by the process of multiplication. The example that required the services of an expert in duplation is within the capability of most children because of the new numerals. The method used in multiplication involves separating one of the numbers to be multiplied into units and tens, multiplying each of these quantities by the second number, and adding the partial products:

$$\begin{array}{r} 32 \\ \underline{17} \\ 224 \\ \underline{32\phantom{0}} \\ 544 \end{array}$$

The French mathematician Laplace commented on the Hindu-Arabic numerals by saying:

> It is India that gave us the ingenious method of expressing all numbers by means of ten symbols, each symbol receiving a value of position as well as an absolute value; a pro-

> found and important idea which appears so simple to us now that we ignore its true merit. But its very simplicity, the great ease which it has lent to all computations, puts our arithmetic in the first rank of useful inventions. 34

The elements of the unambiguous place notation that we use today were developed by the inhabitants of Mesopotamia, India, and the Yucatan peninsula. The last step, recognition of zero as a number by the public, took place only in India.

We have seen the development of cardinal and ordinal number concepts, the creation of the number word, the grouping of numbers by using a number base, the use of simple number symbols, the discovery of the principle of place notation, the development of a symbol for the absence of a digit, and the recognition of zero as a number with which computations may be made. Each of these steps was important in producing our arithmetic, the tool of the housewife as well as the scientist.

## Footnotes

1/ Tobias Dantzig, Number, the Language of Science (London: George Allen & Unwin Ltd., 1930), pp. 1-3.

2/ Edna E. Kramer, The Main Stream of Mathematics (New York: Oxford University Press, 1951), p. 8.

3/ At the Tropical Research Institute of the University of El Salvador.

4/ Waldemar Kaempffert, "Insects That Recognize Numbers," New York Times, April 4, 1954, sec. 4, p. 9E.

5/ Dantzig, op. cit., p. 4.

6/ Lee Emerson Boyer, Mathematica - a Historical Development (New York: Henry Holt & Co., 1946), chap. I passim.

7/ Webster's New Collegiate Dictionary, 2 nd ed. (Springfield, Mass.: G. & C. Merriam Co., 1953).

8/ Boyer, op. cit., p. 14.

9/ From studies made by Franz Boas, as discussed by Dantzig, op. cit., p. 6, and Roy Dubisch, The Nature of Number, An Approach to Basic Ideas of Modern Mathematics (New York: Ronald Press Co., 1952), p. 7.

10/ Dantzig, op. cit., p. 6.

11/ Ibid.; Dubisch, loc. cit.

12/ Levi Leonard Conant, The Number Concept, Its Origin and Development (New York: The Macmillan Co., 1896), pp. 72, 73.

13/ Dantzig, op. cit., p. 12.

14/ Conant, op. cit., chaps. III and IV passim.

15/ Dantzig, op. cit., p. 5.

16 Conant, op. cit., pp. 72, 73.

17 Jean Piaget, "How Children Form Mathematical Concepts," Scientific American, Vol. 189, no. 5 (November, 1953), p. 74.

18 Webster's New Collegiate Dictionary, 2nd ed.

19 The Encyclopedia Britannica (14th ed.; New York, London: The Encyclopedia Britannica Co., 1929), XVI, 610.

20 A. C. Haddon, "Western Tribes of the Torres Straits," Journal of the Anthropological Institute (1889), p. 303, as cited by Conant, op. cit., p. 105. Also, Dantzig, op. cit., p. 18, and Kramer, op. cit., p. 12.

21 Dubisch, op. cit., p. 11; Dantzig, op. cit., p. 18.

22 Encyclopedia Britannica, loc. cit.

23 F. E. Andrews, New Numbers (New York: Harcourt Brace & Co., 1935), p. 23.

24 Ibid., passim.

25 Dantzig, op. cit., p. 21.

26 Encyclopedia Britannica, loc. cit.

27 Ibid.

28 James Hastings, ed., Encyclopedia of Religion and Ethics (New York: Chas. Scribner's Sons, 1951), IX, 406.

29 Kramer, op. cit., p. 20.

30 David E. Smith, History of Mathematics (Boston: Ginn and Company, 1925), II, 54-58.

31 Ibid., pp. 43-45; H. S. Kaltenborn, Meaningful Mathematics (New York; Prentice-Hall, 1951), p. 11.

32 Smith, *op. cit.*, pp. 156-196.

33 Dantzig, *op. cit.*, p. 29.

34 *Ibid.*, p. 19.

## Glossary

<u>abacus</u> ........................ a simple calculating instrument.

<u>base</u> .......................... last simple (uncompounded) number.

<u>binary system</u> ................. a number system using two basic numbers.

<u>cardinal numbers</u> .............. numerals that resemble tally marks; imply no counting.

<u>decimal</u> ....................... pertaining to a number system that uses ten basic numbers; a means of representing fractions without using denominators - <u>e. g.</u>, decimal fractions.

<u>duodecimal</u> .................... pertaining to a number system that uses twelve basic numbers.

<u>duplation</u> ..................... multiplication performed by successive doubling and addition of the products.

<u>mediation</u> ..................... division performed by repeated halving.

<u>ordinal numbers</u> ............... a set of numbers that are related only by their place in a sequence; imply counting.

<u>one-to-one correspondence</u> ..... indicating number by matching.

<u>prime</u> ......................... a number that is evenly divisible only by itself and one (7, 13, 23).

<u>quaternary</u> .................... pertaining to a number system that uses four basic numbers.

<u>quinary</u> ...................... pertaining to a number system that uses five basic numbers.

<u>sexagesimal</u> .................. pertaining to a number system that uses sixty simple numbers (no pure sexagesimal system has been used).

<u>tertiary</u> ..................... pertaiming to a number system that uses three basic numbers.

<u>vigesimal</u> .................... pertaining to a number system that uses twenty independent number words.

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