

CAUSAL INFERENCE: LOGICAL FOUNDATION AND NEW RESULTS

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3-LEVEL HIERARCHY OF CAUSAL MODELS

1. Probabilistic Knowledge $P(y | x)$
Bayesian networks, graphical models
2. Interventional Knowledge $P(y | do(x))$
Causal Bayesian Networks (CBN)
(Agnostic graphs, manipulation graphs)
3. Counterfactual Knowledge $P(Y_x = y)$
Structural equation models, physics,
functional graphs

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BAYESIAN NETWORKS

Definition

Algebraic Representation

Graphical Representation

(d-separation = conditional independency)



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CAUSAL BAYESIAN NETWORK (CBN) (Agnostic graphs, manipulation graphs)

Definition

Let $P_x(v)$ stand for $P(v | do(X=x))$

G is a CBN relative to P_x^* iff:

- i. $P_x(v)$ is Markov relative to G ;
- ii. $P_x(v_i)=1$ for all $V_i \in X$ if v_i agrees with $X=x$.
- iii. $P_x(v_i | pa_i) = P(v_i | pa_i)$ for all $V_i \notin X$ whenever pa_i agrees with $X=x$, i.e., each $P(v_i | pa_i)$ remains INVARIANT to interventions not involving V_i .

Algebraic representation

$$P_x(v) = \prod_{\{i|V_i \notin X\}} P(v_i | pa_i)$$

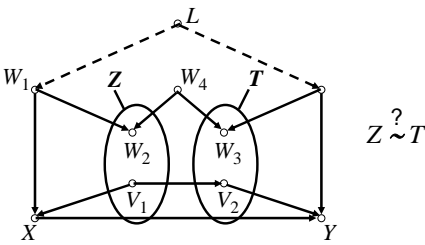
(Truncated product, G -computation, manipulation theorem)

Graphical representation

Surgery: remove incoming arrows to X .

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CONFOUNDING EQUIVALENCE WHEN TWO MEASUREMENTS ARE EQUALLY VALUABLE



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CONFOUNDING EQUIVALENCE WHEN TWO MEASUREMENTS ARE EQUALLY VALUABLE

Definition:

T and Z are c -equivalent if

Definition (Markov boundary):

Markov boundary S_m of S (relative to X) is the minimal subset of S that d -separates X from all other members of S .

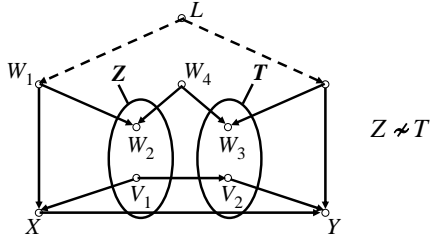
Theorem (Pearl and Paz, 2009)

Z and T are c -equivalent iff

1. $Z_m = T_m$, or
2. Z and T are admissible (i.e., satisfy the back-door condition)

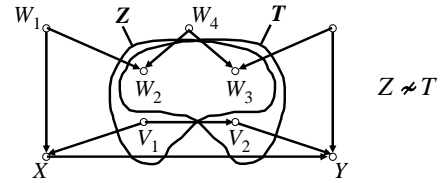
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CONFOUNDING EQUIVALENCE
WHEN TWO MEASUREMENTS ARE
EQUALLY VALUABLE



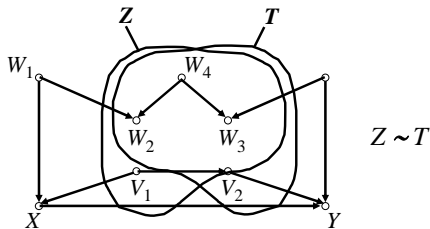
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CONFOUNDING EQUIVALENCE
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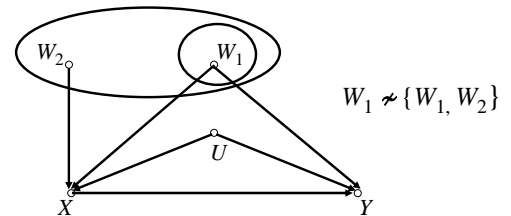
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CONFOUNDING EQUIVALENCE
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BIAS AMPLIFICATION
BY INSTRUMENTAL VARIABLES



- Adding W_2 to Propensity Score increases bias (if such exists) (Wooldridge, 2009)
- In linear systems – always
- In non-linear systems – almost always (Pearl, 2010)
- Outcome predictors are safer than treatment predictors

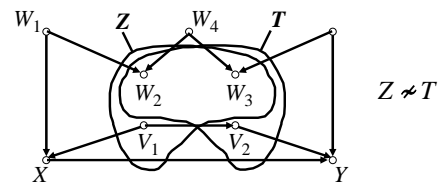
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RAMIFICATION OF THE
MEDIATION FORMULA

- DE should be averaged over mediator levels,
 IE should NOT be averaged over exposure levels.
- $TE-DE$ need not equal IE
 $TE-DE$ = proportion for whom mediation is necessary
 IE = proportion for whom mediation is sufficient
- $TE-DE$ informs interventions on indirect pathways
 IE informs intervention on direct pathways.

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CONFOUNDING EQUIVALENCE
WHEN TWO MEASUREMENTS ARE
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