

THE SYMBIOTIC APPROACH TO CAUSAL INFERENCE

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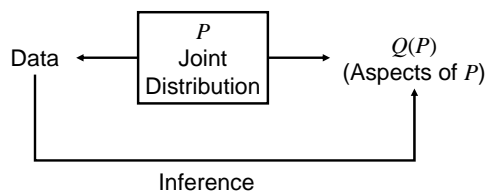
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OUTLINE

- Inference: Statistical vs. Causal, distinctions, and mental barriers
- Unified conceptualization of counterfactuals, structural-equations, and graphs
- The logical equivalence of SEM and potential outcomes
- How to use the best features of both
- Direct and Indirect effects
- The Mediation Formula and its ramifications

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TRADITIONAL STATISTICAL INFERENCE PARADIGM

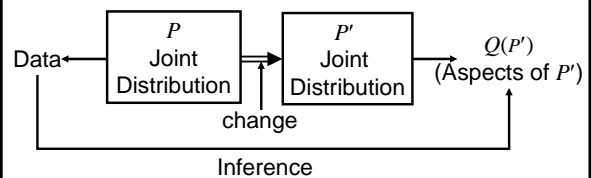


e.g.,
Infer whether customers who bought product A
would also buy product B.
 $Q = P(B | A)$

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FROM STATISTICAL TO CAUSAL ANALYSIS: 1. THE DIFFERENCES

Probability and statistics deal with static relations



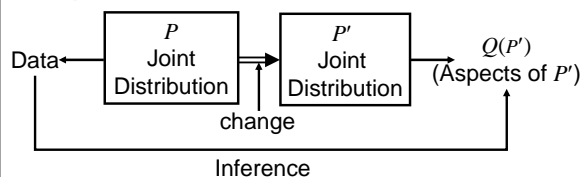
What happens when P changes?

e.g.,
Infer whether customers who bought product A
would still buy A if we were to double the price.

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FROM STATISTICAL TO CAUSAL ANALYSIS: 1. THE DIFFERENCES

What remains invariant when P changes say, to satisfy
 $P'(price=2)=1$



Note: $P'(v) \neq P(v | price = 2)$

P does not tell us how it ought to change
e.g. Curing symptoms vs. curing diseases
e.g. Analogy: mechanical deformation

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FROM STATISTICAL TO CAUSAL ANALYSIS: 1. THE DIFFERENCES (CONT)

1. Causal and statistical concepts do not mix.

CAUSAL

Spurious correlation
Randomization / Intervention
Confounding / Effect
Instrumental variable
Exogeneity / Ignorability
Mediation

STATISTICAL

Regression
Association / Independence
"Controlling for" / Conditioning
Odd and risk ratios
Collapsibility / Granger causality
Propensity score

2.

3.

4.

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FROM STATISTICAL TO CAUSAL ANALYSIS: 2. MENTAL BARRIERS

- Causal and statistical concepts do not mix.

CAUSAL	STATISTICAL
Spurious correlation	Regression
Randomization / Intervention	Association / Independence
Confounding / Effect	"Controlling for" / Conditioning
Instrumental variable	Odd and risk ratios
Exogeneity / Ignorability	Collapsibility / Granger causality
Mediation	Propensity score
- No causes in – no causes out (Cartwright, 1989)

statistical assumptions + data } ⇒ causal conclusions
causal assumptions }
- Causal assumptions cannot be expressed in the mathematical language of standard statistics.
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FROM STATISTICAL TO CAUSAL ANALYSIS: 2. MENTAL BARRIERS

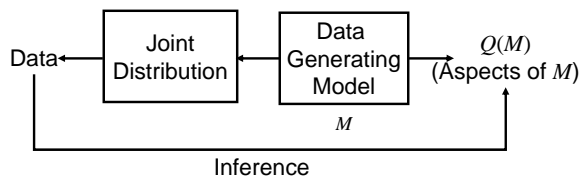
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CAUSAL	STATISTICAL
Spurious correlation	Regression
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Instrumental variable	Odd and risk ratios
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- No causes in – no causes out (Cartwright, 1989)

statistical assumptions + data } ⇒ causal conclusions
causal assumptions }
- Causal assumptions cannot be expressed in the mathematical language of standard statistics.
- Non-standard mathematics:
 - Structural equation models (Wright, 1920; Simon, 1960)
 - Counterfactuals (Neyman-Rubin (Y_x), Lewis ($x \boxrightarrow Y$))

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THE STRUCTURAL MODEL PARADIGM



M – Invariant strategy (mechanism, recipe, law, protocol) by which Nature assigns values to variables in the analysis.

“Think Nature, not experiment!”

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STRUCTURAL CAUSAL MODELS

Definition: A structural causal model is a 4-tuple $\langle V, U, F, P(u) \rangle$, where

- $V = \{V_1, \dots, V_n\}$ are endogeneous variables
- $U = \{U_1, \dots, U_m\}$ are background variables
- $F = \{f_1, \dots, f_n\}$ are functions determining V ,
 $v_i = f_i(v, u)$ e.g., $y = \alpha + \beta x + u_Y$
- $P(u)$ is a distribution over U

$P(u)$ and F induce a distribution $P(v)$ over observable variables

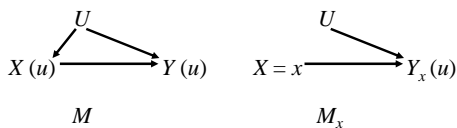
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CAUSAL MODELS AND COUNTERFACTUALS

Definition:

The sentence: “ Y would be y (in unit u), had X been x ,” denoted $Y_x(u) = y$, means:

The solution for Y in a mutilated model M_x (i.e., the equations for X replaced by $X = x$) with input $U = u$, is equal to y .



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The Fundamental Equation of Counterfactuals:

$$\boxed{Y_x(u) = Y_{M_x}(u)}$$

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- Joint probabilities of counterfactuals:

$$P(Y_x = y, Z_w = z) = \sum_{u: Y_x(u)=y, Z_w(u)=z} P(u)$$

In particular:

$$P(y | do(x)) \triangleq P(Y_x = y) = \sum_{u: Y_x(u)=y} P(u)$$

$$PN(Y_{x'} = y' | x, y) = \sum_{u: Y_{x'}(u)=y'} P(u | x, y)$$

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TWO PARADIGMS FOR CAUSAL INFERENCE

Observed: $P(X, Y, Z, \dots)$
Conclusions needed: $P(Y_x = y), P(X_y = x | Z = z) \dots$

How do we connect observables, X, Y, Z, \dots to counterfactuals Y_x, X_z, Z_y, \dots ?

<p><u>N-R model</u> Counterfactuals are primitives, new variables Super-distribution $P^*(X, Y, \dots, Y_x, X_z, \dots)$ X, Y, Z constrain Y_x, Z_y, \dots</p>	<p><u>Structural model</u> Counterfactuals are derived quantities Subscripts modify the model and distribution $P(Y_x = y) = P_{M_x}(Y = y)$</p>
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ARE THE TWO PARADIGMS EQUIVALENT?

- Yes (Galles and Pearl, 1998; Halpern 1998)
- In the N-R paradigm, Y_x is defined by consistency:

$$Y = xY_1 + (1-x)Y_0$$
- In SCM, consistency is a theorem.
- Moreover, a theorem in one approach is a theorem in the other.

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THE FIVE NECESSARY STEPS OF CAUSAL ANALYSIS

Define: Express the target quantity Q as a function $Q(M)$ that can be computed from any model M .

Assume: Formulate causal assumptions A using some formal language.

Identify: Determine if Q is identifiable given A .

Estimate: Estimate Q if it is identifiable; approximate it, if it is not.

Test: Test the testable implications of A (if any).

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THE FIVE NECESSARY STEPS FOR EFFECT ESTIMATION

Define: Express the target quantity Q as a function $Q(M)$ that can be computed from any model M .

$$ATE \triangleq E(Y | do(x_1)) - E(Y | do(x_0))$$

Assume: Formulate causal assumptions A using some formal language.

Identify: Determine if Q is identifiable given A .

Estimate: Estimate Q if it is identifiable; approximate it, if it is not.

Test: Test the testable implications of A (if any).

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FORMULATING ASSUMPTIONS THREE LANGUAGES

- English: Smoking (X), Cancer (Y), Tar (Z), Genotypes (U)
- Counterfactuals: $Z_x(u) = Z_{yx}(u)$,
 $X_y(u) = X_{zy}(u) = X_z(u) = X(u)$,
 $Y_z(u) = Y_{zx}(u)$,
 $Z_x \perp\!\!\!\perp \{Y_z, X\}$
 Not too friendly:
 consistent? complete? redundant? arguable?
- Structural:


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                graph LR
                X --> Z
                Z --> Y
                X -.-> Y
            
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IDENTIFYING CAUSAL EFFECTS IN POTENTIAL-OUTCOME FRAMEWORK

Define: Express the target quantity Q as a counterfactual formula, e.g., $E(Y(1) - Y(0))$

Assume: Formulate causal assumptions using the distribution:

$$P^* = P(X | Y, Z, Y(1), Y(0))$$

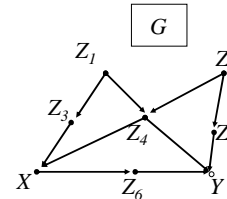
Identify: Determine if Q is identifiable using P^* and $Y = x Y(1) + (1 - x) Y(0)$.

Estimate: Estimate Q if it is identifiable; approximate it, if it is not.

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IDENTIFICATION IN SCM

Find the effect of X on Y , $P(y|do(x))$, given the causal assumptions shown in G , where Z_1, \dots, Z_k are auxiliary variables.

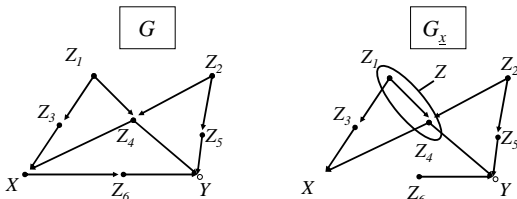


Can $P(y|do(x))$ be estimated if only a subset, Z , can be measured?

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ELIMINATING CONFOUNDING BIAS THE BACK-DOOR CRITERION

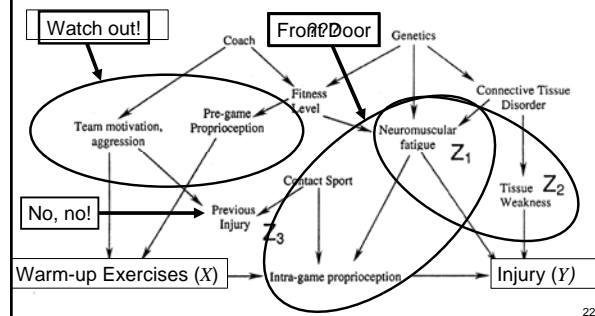
$P(y | do(x))$ is estimable if there is a set Z of variables such that Z d -separates X from Y in G_x .



Moreover, $P(y | do(x)) = \sum_z P(y | x, z) P(z) = \sum_z \frac{P(x, y, z)}{P(x | z)}$
("adjusting" for Z) \rightarrow Ignorability

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EFFECT OF WARM-UP ON INJURY (After Shrier & Platt, 2008)



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GRAPHICAL – COUNTERFACTUALS SYMBIOSIS

Every causal graph expresses counterfactual assumptions, e.g., $X \rightarrow Y \rightarrow Z$

1. Missing arrows $Y \leftarrow Z$ $Y_{x,z}(u) = Y_x(u)$

2. Missing arcs $Y \leftarrow Z$ $Y_x \perp\!\!\!\perp Z_y$

consistent, and readable from the graph.

- Express assumption in graphs
- Derive estimands by graphical or algebraic methods

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EFFECT DECOMPOSITION (direct vs. indirect effects)

- Why decompose effects?
- What is the definition of direct and indirect effects?
- What are the policy implications of direct and indirect effects?
- When can direct and indirect effect be estimated consistently from experimental and nonexperimental data?

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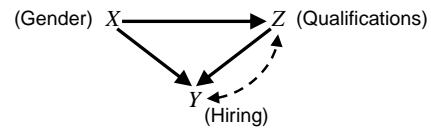
WHY DECOMPOSE EFFECTS?

1. To understand how Nature works
2. To comply with legal requirements
3. To predict the effects of new type of interventions:
Signal routing, rather than variable fixing

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LEGAL IMPLICATIONS OF DIRECT EFFECT

Can data prove an employer guilty of hiring discrimination?



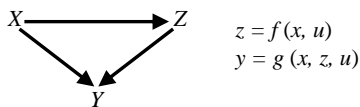
What is the direct effect of X on Y?

$E(Y | do(x_1), do(z)) - E(Y | do(x_0), do(z))$
(averaged over z) Adjust for Z? No! No!

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NATURAL INTERPRETATION OF AVERAGE DIRECT EFFECTS

Robins and Greenland (1992) – “Pure”



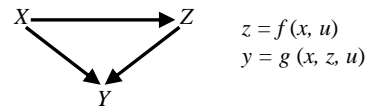
Natural Direct Effect of X on Y: $DE(x_0, x_1; Y)$
The expected change in Y, when we change X from x_0 to x_1 and, for each u , we keep Z constant at whatever value it attained before the change.

$$E[Y_{x_1 Z_{x_0}} - Y_{x_0}]$$

In linear models, $DE =$ Controlled Direct Effect

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DEFINITION OF INDIRECT EFFECTS



Indirect Effect of X on Y: $IE(x_0, x_1; Y)$
The expected change in Y when we keep X constant, say at x_0 , and let Z change to whatever value it would have attained had X changed to x_1 .

$$E[Y_{x_0 Z_{x_1}} - Y_{x_0}]$$

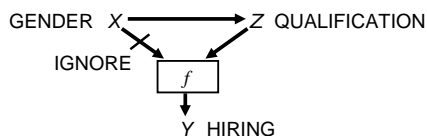
In linear models, $IE = TE - DE$

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POLICY IMPLICATIONS OF INDIRECT EFFECTS

What is the indirect effect of X on Y?

The effect of Gender on Hiring if sex discrimination is eliminated.



Blocking a link – a new type of intervention

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MEDIATION FORMULAS

1. The natural direct and indirect effects are identifiable in models with no unobserved confounders, and are given by:

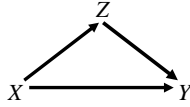
$$DE = \sum_z [E(Y | do(x_1, z)) - E(Y | do(x_0, z))] P(z | do(x_0))$$

$$IE = \sum_z E(Y | do(x_0, z)) [P(z | do(x_1)) - P(z | do(x_0))]$$

2. Applicable to linear and non-linear models, continuous and discrete variables, regardless of distributional form.

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MEDIATION FORMULAS IN UNCONFOUNDED MODELS



$$DE = \sum_z [E(Y | x_1, z) - E(Y | x_0, z)] P(z | x_0)$$

$$IE = \sum_z [E(Y | x_1, z) [P(z | x_1) - P(z | x_0)]]$$

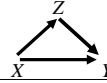
$$TE = E(Y | x_1) - E(Y | x_0)$$

IE = Fraction of responses explained by mediation

$TE - DE$ = Fraction of responses owed to mediation

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COMPUTING THE MEDIATION FORMULA



	X	Z	Y	$E(Y x,z) = g_{xz}$	$E(Z x) = h_x$
n_1	0	0	0	$\frac{n_2}{n_1 + n_2} = g_{00}$	$\frac{n_3 + n_4}{n_1 + n_2 + n_3 + n_4} = h_0$
n_2	0	0	1		
n_3	0	1	0	$\frac{n_4}{n_3 + n_4} = g_{01}$	
n_4	0	1	1		
n_5	1	0	0	$\frac{n_6}{n_5 + n_6} = g_{10}$	$\frac{n_7 + n_8}{n_5 + n_6 + n_7 + n_8} = h_1$
n_6	1	0	1		
n_7	1	1	0	$\frac{n_8}{n_7 + n_8} = g_{11}$	
n_8	1	1	1		

$$DE = (g_{10} - g_{00})(1 - h_0) + (g_{11} - g_{01})h_0$$

$$IE = (h_1 - h_0)(g_{01} - g_{00})$$

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RAMIFICATIONS OF THE MEDIATION FORMULA

- Effect measures should not be defined in terms of *ML* estimates of regression coefficients (e.g. polynomials, logistic, or probit,) but in terms of population distributions.
- The difference and product heuristics resemble $TE-DE$ and IE , but:
 - DE should be averaged over mediator levels,
 - IE should NOT be averaged over exposure levels.
- $TE-DE$ need not equal IE
 - $TE-DE$ = proportion for whom mediation is necessary
 - IE = proportion for whom mediation is sufficient.
- $TE-DE$ informs interventions on indirect pathways
 IE informs intervention on direct pathways.

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CONCLUSIONS

IT IS DOWNTON CAUSAL DATA SIMPLICE

on an explicit causal structure that is defensible on scientific grounds.

- Formal basis for causal and counterfactual inference (complete)
- Unification of the graphical, potential-outcome and structural equation approaches (Aristotle 384-322 B.C.)

Friendly and formal solutions to century-old problems and confusions.
From Charlie Poole

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QUESTIONS???

They will be answered

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