

Final Exam

CS262Z, Spring 2007

University of California, Los Angeles

Question 1

Show that, in linear models, the counterfactual expectation $E(Y_x|Y = y', X = x')$ is identified whenever the total effect of X on Y is identified. Specifically, prove that:

$$E(Y_x|Y = y', X = x') = y' + t(x - x')$$

where t is given by:

$$t = [E(Y_x) - E(Y_{x'})]/(x - x')$$

Question 2

Halpern and Pearl [2000] propose the following definition for actual cause within the framework of structural causal models:

Definition 1. Let $M = (U, V, F)$ be a causal model. Let $X \subseteq V$, $Y \subseteq V$. $X = x$ is an actual cause of $Y = y$ in a causal model M under a specific setting \mathbf{u} of the background variables U iff the following three conditions hold:

(AC1) $X(\mathbf{u}) = x$ and $Y(\mathbf{u}) = y$.

(AC2) There exists $W \subseteq V \setminus X$ and values $x' \in \text{Dom}(X)$ and $w \in \text{Dom}(W)$ such that:

(a) $Y_{x'w}(\mathbf{u}) \neq y$.

(b) $Y_{xw}(\mathbf{u}) = y$.

(c) $Y_{xwz}(\mathbf{u}) = y$, for all $Z \subseteq V \setminus (X \cup W)$ such that $z = Z(\mathbf{u})$.

(AC3) X is minimal; no subset of X satisfies conditions AC1 and AC2.

Use this definition of actual cause to answer the following questions (note that for simplicity you can assume the background variable set to be empty for each structural model formulation). If you answer “yes” to a question, show that all parts of the definition are satisfied. If you answer “no” to a question, show that one of the conditions of the definition is violated.

1. *Example 1.* (Loanshark) Larry the Loanshark contemplates lurking outside of Fred’s workplace to cut off his finger, as a warning to him to repay his loan quickly. Something comes up, however, so he does not do so. That same day, Fred has his finger severed by a machine at the factory. He is rushed to the hospital, where the finger is reattached, so if Larry had shown up, he would have missed Fred. At day’s end, Fred’s finger is functional, which would not have been true had Larry shown up and Fred not had his accident.

Formulate this scenario as a structural causal model and then use the same model to answer all of the following:

- (a) Is Larry's failure to lurk outside of Fred's workplace an actual cause of Fred's finger being functional at the end of the day, according to the definition above?
 - (b) Is Fred's accident at the factory an actual cause of Fred's finger being functional at the end of the day, according to the definition above?
 - (c) Is Fred's accident at the factory AND getting rushed to the hospital (conjunction of events) an actual cause of Fred's finger being functional at the end of the day, according to the definition above?
2. *Example 2.* (Bomb – due to Yablo) Billy puts a bomb under Suzy's chair. Later, Suzy notices the bomb and flees the room, before the chair explodes. Still later, Suzy has a prearranged medical checkup and is pronounced healthy. Formulate this scenario as a structural causal model and then use the same model to answer all of the following:
- (a) Is the chair exploding an actual cause of Suzy being pronounced healthy, according to the definition above?
 - (b) Is Suzy fleeing the room an actual cause of Suzy being pronounced healthy, according to the definition above?
 - (c) Is Billy planting the bomb under Suzy's chair an actual cause of Suzy being pronounced healthy, according to the definition above?