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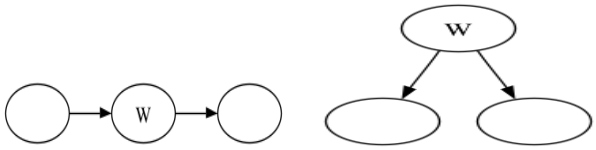
These are my notes for cs262a, reasoning with partial beliefs. The notes are taken from Professor Adnan Darwiche's (soon to be published) book, "Modeling and Reasoning with Bayesian Networks" and his in class lectures.

# 1 d-separation




| | **sequential** | **divergent** | **convergent** |
|---|---|---|---|
| **open** | $w \notin Z$ | $w \notin Z$ | $w \in Z$ or descendent$(w) \in Z$ |
| **closed** | $w \in Z$ | $w \in Z$ | $w \notin Z$ and descendent$(w) \notin Z$ |

# 2 Properties of Probabilistic Independence

## 2.1 General

- $\boldsymbol{I(X, \text{Parents}(X), \text{Non\_Descendents}(X))}$ - $\boldsymbol{X}$ is independent of it's non-descendents given it's parents.

## 2.2 Graphoid Axioms

- **Symmetry:**
  $\boldsymbol{I(X, Z, Y) \iff I(Y, Z, X)}$ - If learning $\boldsymbol{y}$ does not influence our belief in $\boldsymbol{x}$, then learning $\boldsymbol{x}$ does not influence our belief in $\boldsymbol{y}$ either.
- **Decomposition:**
  $\boldsymbol{I(X, Z, Y \cup W) \implies I(X, Z, Y)}$ & $\boldsymbol{I(X, Z, W)}$ - If learning $\boldsymbol{yw}$ does not influence our belief in $\boldsymbol{x}$, then learning $\boldsymbol{y}$ or $\boldsymbol{w}$ alone will not influence our belief in $\boldsymbol{x}$.
- **Weak Union:**
  $\boldsymbol{I(X, Z, Y \cup W) \implies I(X, Z \cup Y, W)}$ - If the information $\boldsymbol{yw}$ is not relevant to our belief in $\boldsymbol{x}$, then the partial information $\boldsymbol{y}$ will not make the rest of the information, $\boldsymbol{w}$, relevant.
- **Contraction:**
  $\boldsymbol{I(X, Z, Y)}$ & $\boldsymbol{I(X, Z \cup Y, W) \implies I(X, Z, Y \cup W)}$ - If after learning the irrelevant information $\boldsymbol{y}$ the information $\boldsymbol{w}$ is found to be irrelevant to our belief in $\boldsymbol{x}$, the the combined information $\boldsymbol{yw}$ must have been irrelevant from the beginning.

## 2.3 For strictly positive probability distributions

- **Intersection:**
  $\boldsymbol{I(X, Z \cup W, Y)}$ & $\boldsymbol{I(X, Z \cup Y, W) \implies I(X, Z, Y \cup W)}$ - If information $\boldsymbol{y}$ is irrelevant given $\boldsymbol{w}$ and information $\boldsymbol{w}$ is irrelevant given $\boldsymbol{y}$, then the combined information $\boldsymbol{yw}$ is irrelevant to start with.