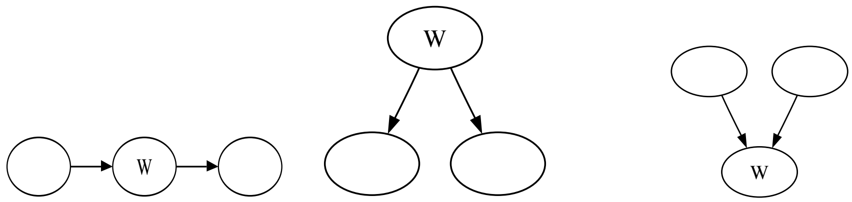
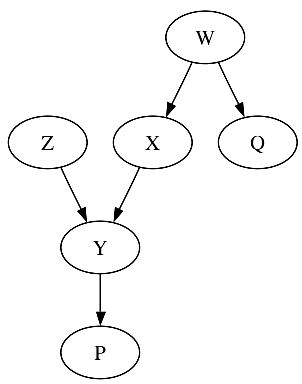
d-separation

d-separation is a graphical test of independence between variables in a directed acyclic graph. This is a very useful tool for working with Bayesian networks. Given two sets of variables A and B, we test if they are independent conditioned on a set Z of variables by checking all paths between each variable in A and each variable in B. We say that A is independent of B given Z $(A \perp\!\!\!\perp B|Z)$ is all paths between each variable in A and B are closed when we condition on (or in other words observe) Z. If any path is open, we cannot claim independence but also cannot claim dependence. We would have to examine the conditional probability tables to verify the independence claims if there is no d-separation. A path is closed if there is any sequence of vertices and edges on it that are closed according to the following chart:



	sequential	**divergent**	**convergent**
open	$W \notin Z$	$W \notin Z$	$W \in Z$ or descendent$(w) \in Z$
closed	$W \in Z$	$W \in Z$	$W \notin Z$ and descendent$(w) \notin Z$

Example



On this graph, we can say the following (this is not an exhaustive list):

- $\boldsymbol{(Q \perp\!\!\!\perp X, Y, Z, P|W)}$: $Q \to W \to X$ is a divergent path that is closed since we condition on W.
- $\boldsymbol{(Z \perp\!\!\!\perp X, W, Q|\varnothing)}$: $Z \to Y \leftarrow X$ is a closed convergent path since we do not condition on Y or it's descendent P. Likewise:
- $\boldsymbol{(Z \not\perp\!\!\!\perp X, W, Q|P)}$: $Z \to Y \leftarrow X$ is an open convergent path since we condition on P, a descendent of Y.
- $\boldsymbol{(Z, Y, P \perp\!\!\!\perp W, Q|X)}$: $W \to X \to Y$ is a closed sequential path since we condition on X. Likewise
- $\boldsymbol{(Z, Y, P \not\perp\!\!\!\perp W, Q|\varnothing)}$: $W \to X \to Y$ is an open sequential path since we do not condition on X.