Homework 3 – Problem 4

a) Just do simple algrebra

b)

_ Sort all the element in nondecreasing order. This will take O(nlogn)

$x_1\ x_2\ x_3\ \ldots..\ x_n$ (where $x_I < x_{I+1}$)

$w_1\ w_2\ w_3\ \ldots.\ w_n$

_ Initialize sum=0;

_ Keep adding w_I (from left to right) until we meet the first w_k that make sum > 0.5

_ That w_k is the weighted median

c) Define our new select function as:

S' (beg_index, end_index, w1, w2): This function find the weighted median k such that:

$\sum w_I < w1$ (where beg_index < i < k)

$\sum w_I < w2$ (where k < i < end_index)

Step 1: Find pivot x_k (normal median) using the normal select function (in the book)

Step2: sum = $\sum w_I$ (i < k)

Step 3:

if (sum > w1), that means the weighted median is in the left of x_k. Recursively do the S' function:
S' (beg_index, k - 1, w1, sum – w2);

if (sum < w1), that means the weighted median is in the right of x_k. Recursively do the S' function:
S' (k + 1, end_index, w1 - sum, w2);

Base case: If (beg_index == end_index) => weighted median is x_{beg_index}

d)

$K(y) = \sum w_I\, d(y, p_I)$

Pick a p_j in the left of the weighted median p_k:

$K(p_j) = \sum w_I |p_j - p_I|$
$= \sum w_I (p_j - p_I) + \sum w_I' (p_j - p_I')$ (where $p_I' < p_j < p_I$)

If we move the point toward the weighted median a distance ε

$K(p_j + w) - K(p_j) = \sum w_I |p_j - p_I|$
$= \sum w_I\, \varepsilon - \sum w_I'\, \varepsilon$ (where $p_I' < p_j < p_I$)
$= \varepsilon (\sum w_I - \sum w_I')$

Since the p_j is in the left of the weighted median, $(\sum w_I - \sum w_I') < 0$

Therefore $K(p_j + w) < K(p_j)$

Prove the same thing when p_j is in the right of p_k

Therefore in order to minimize K, we have to chose the wiehgted median.

e) $K(p) = \sum w_I (|p_x - p_{I\,x}| + |p_y - p_{I\,y}|)$
$= \sum w_I |p_x - p_{I\,x}| + \sum w_I |p_y - p_{I\,y}|$

Use part c to find the weighted median p_z to minimize $\Sigma\ w_I\ |p_x - p_{I\ x}|$
And then use part c again to find the weighted median p_y to minimize $\Sigma\ w_I\ |p_y - p_{I\ y}|$

The point will be $p(p_x, p_y)$.