Problem Set #3 - Problem 17-2

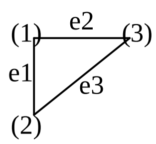
a) Matroid: G is an undirected graph, G = (V, E)

1) S = E which is a finite set

2) *I* is hereditary because removing an edge from an acyclic set of edges doesn't create a cycle.

3) Suppose A and B are forests of G and that |B| > |A|; A, B are acyclic sets of edges which
Forest A contains |V| - |A| trees
Forest B contains |V| - |B| trees

Thus forest B has less trees than forest A and therefore there exists some tree T in forest B whose vertices are in two different trees in forest A. Since T is connected, it contains an edge (u, v), such that the vertices u and v are in different trees in forest A. Since the edge (u, v) connects two different trees in forest A, the edge (u, v) can be added to the forest without creating a cycle. This satisfies the exchange property and completes the matroid proof.

b) 

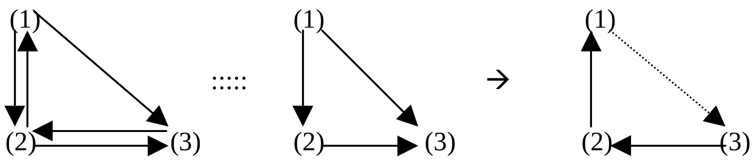
	e1	e2	e3
1	1	1	0
2	1	0	1
3	0	1	1

Using modulo-2 scalar operations, one can see that these set of columns of M are not linearly independent and thus not acyclic. Using these modulo-2 scalar operations, columns are equal and thus can be eliminated which displays linear dependence.

c) Minimum Spanning Tree (or Greedy algorithm on page 348-Cormen)

Greedy (M, *w*)
A ← 0
Sort S[M] into nonincreasing order by weight *w*
For each x ε S[M], taken in nonincreasing order by weight *w*(x)
  Do if A U {x} ε *I* [M]
    Then A ← A U {x}
Return A

d) The exchange condition of a matroid fails for the following directed graph:



Adding the edge (1, 3) from the second diagram to the third fails matroid exchange condition.

e) The same argument as given in part b) can be applied here as well. Or a more intuitive explanation can be seen by examining the exposed edge of a forest and noting that the incidence on the leaf node has rank of 1 and thus only one edge in so its vector will have a zero associated with it. This will give linear independence where as a cycle wouldn't.

f) Well, in parts d) and e) the questions are not dealing with one-to-one correspondences. Also, with directed graphs, it doesn't necessarily have to form a cycle if the columns line up in the incidence matrix. This non-contradictory argument involves the fact that the questions deal with examples of directed graphs and not rules strictly dealing with linear independence. Parts d) and e) don't exclude the fact that there can be a correspondence between linear independence and acyclic edges.