Homework 2, Problem 4

Subquestion (I) We can create a tree according to the hierarchical structure of the company. Each node n of the tree represents one person. We will store two values at each node. One is the maximum sum of conviviality ratings of the subtree rooted at n when n is picked as a guest. Let's denote this value as x_n. The other value is the maximum sum of conviviality ratings of the subtree at n when n is not picked, denoted as $\overline{x_n}$. Now, to solve our problem, we can divide our solution space into two branches: **a**, the president is picked; **b**, the president is not picked. Let us analyze these two situations now.

a: the maximum sum of the conviviality ratings when the president is picked.

$$\mathbf{A} = \sum_{i=1}^{m} \max(x3_i, \overline{x3_i}). \tag{1}$$

where $x3_i$ means the subtree conviviality sum when ith person on the third level of the tree is picked (refer to the definition above), and $\overline{x3_i}$ means the sum when that person is not picked. Basically, it is summing through every subtree on the third level and find the maximum possible.

b: the maximum sum of the conviviality ratings when the president is not picked.

$$\mathbf{B} = \sum_{i=1}^{m} \max(x2_i, \overline{x2_i}). \tag{2}$$

where $x2_i$ means the subtree sum when ith person on the second level is picked (refer to the definition above), and $\overline{x2_i}$ means that person is not picked. Basically, it is summing through every subtree rooted on the second level and find the maximum possible.

Now we will build a table based on the tree from bottom up. The table has T columns, where T represents the total number of people in the company numbered from 1 to T. It has two rows - row 0 stores all the subtree sum values when the corrresponding columns (or corresponding persons) are not picked ($\overline{xl_i}$, where l represents the level of the corresponding person who stays in the tree), and row 1 stores the subtree sum values when columns(persons) are picked (xl_i). We start to fill in the table values when we trace the tree from the bottom up. For example, node 3 is the parent of node 1 and 2, and node 1 and 2 are at the bottom of the tree. Suppose their conviviality ratings are - node 1: 0.2, node 2: 0.4 and node 3: 0.1. We can fill in the table as below:

	1	2	3	4	5	6	...
row 0	0	0	0.6				
row 1	0.2	0.4	0.1				

We continue this trend until we reach the top of the tree and every spot on the table is filled up.

We then can calculate A and B from equation 1 and 2 and we use the bigger sum among the two. The running time should be linear. Because to calculate each xl_i and $\overline{xl_i}$, we only need constant number of calculations.

Subquestion (II) To make sure the president is picked, we have to use equation 1 to calculate A, and that will be the final solution.