On the Identification of Causal Effects

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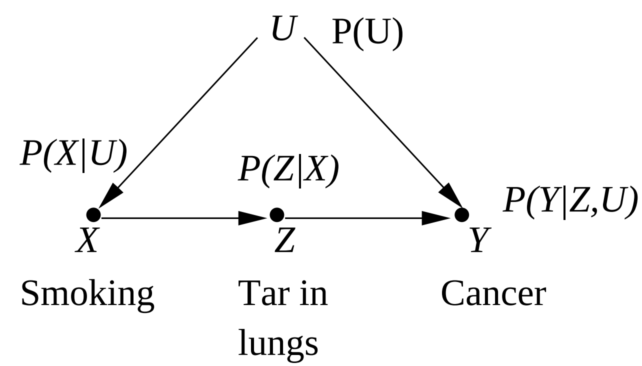
Outline

- Causal Bayesian networks and identification problem
- Previous work
- Identifying causal effects
- Other work
  - Testable implications of causal BNs with hidden variables
  - Characterization of causal BNs

Causal Bayesian Networks

- Causal BNs encode probability distributions

$$P(v_1, \ldots, v_n) = \prod_i P(v_i | pa_i)$$



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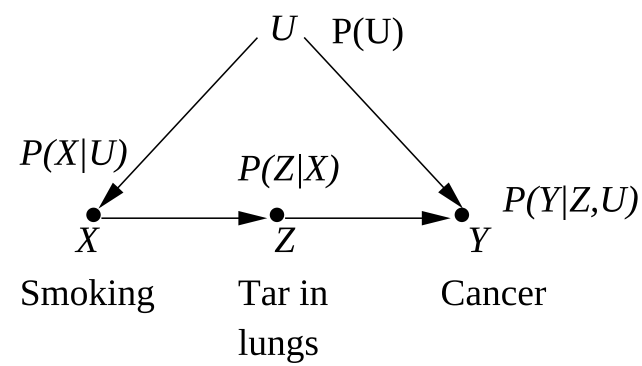

But also

- Encode distributions under external **interventions** (actions).

Interventions

- $do(T = t)$: fixing a set T of variables to some constants $T = t$.

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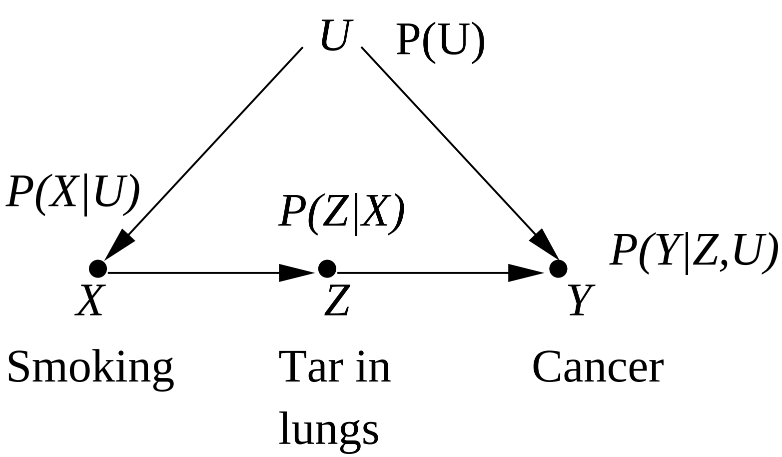

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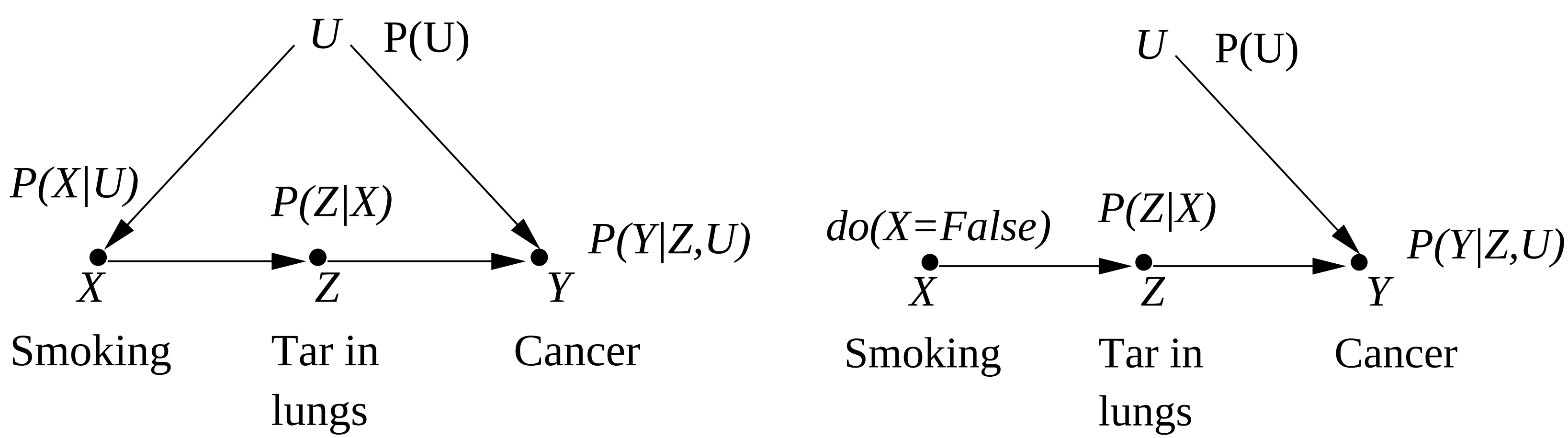

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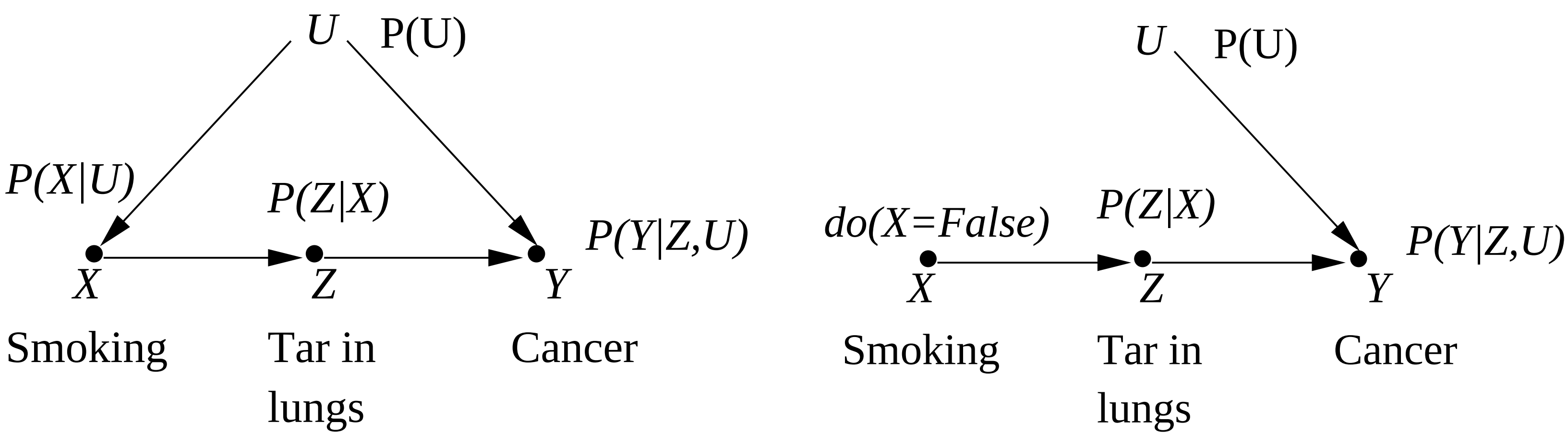

$$P(u,x,z,y) = P(u)P(x|u)P(z|x)P(y|z,u)$$

$$P_{X=False}(u,z,y) = P(u)P(z|X = False)P(y|z,u)$$

Causal Effects

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- Notations in (Pearl 2000) and (Lauritzen 2000):

$$P_t(s) = P(s|do(t)) = P(s|set(t)) = P(s|\hat{t}\,) = P(s||t)$$

Identifying Causal Effects

- Causal BNs can predict causal effects.

$$P(v) = \prod_i P(v_i | pa_i)$$

$$P_t(v) = \prod_{\{i | V_i \notin T\}} P(v_i | pa_i)$$

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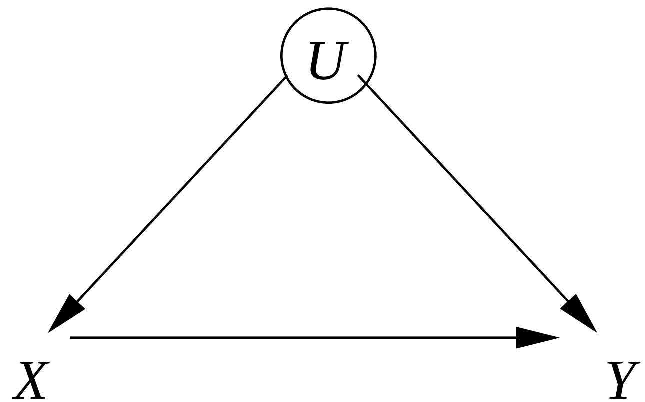
However,

- when only partial causal information is available ...

Identifiability Problem

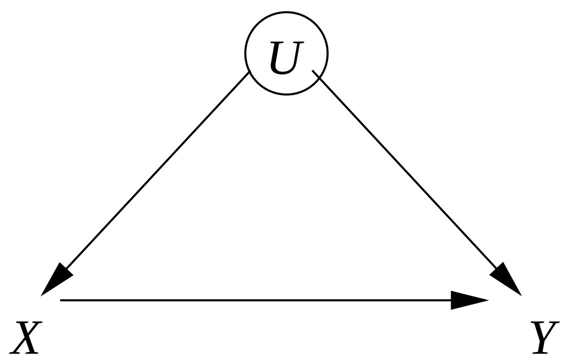
- The presence of unobserved (hidden, latent) variables.



Input: causal graph $+ P(x, y)$.

Can we predict $P_x(y)$?

Identifiability Problem



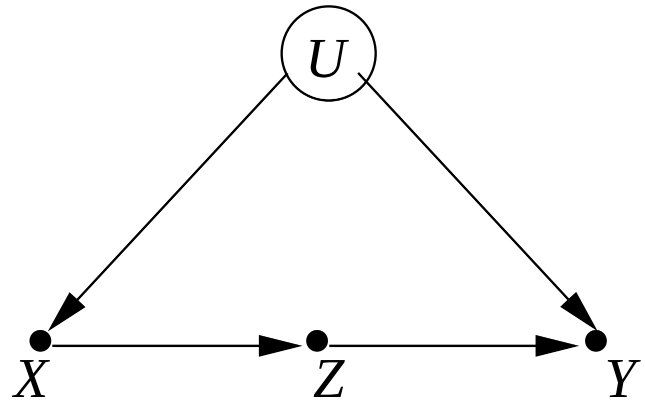
$$P(x,y) = \sum_u P^{M_1}(x|u)P^{M_1}(y|x,u)P^{M_1}(u)$$
$$= \sum_u P^{M_2}(x|u)P^{M_2}(y|x,u)P^{M_2}(u)$$

$$P_x^{M_1}(y) = \sum_u P^{M_1}(y|x,u)P^{M_1}(u)$$

$$P_x^{M_2}(y) = \sum_u P^{M_2}(y|x,u)P^{M_2}(u)$$

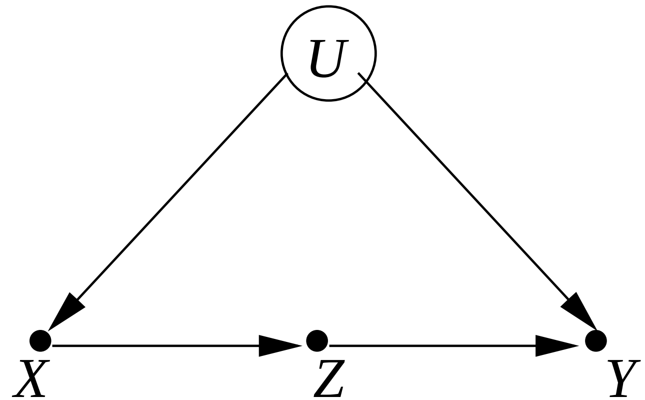
$$P_x^{M_1}(y) \neq P_x^{M_2}(y)$$

Identifiability Problem



Input: causal graph + $P(x, y, z)$.

Identifiability Problem



Input: causal graph $+ P(x, y, z)$.

Output:

$$P_x(y) = \sum_z P(z|x) \sum_{x'} P(y|x', z) P(x')$$

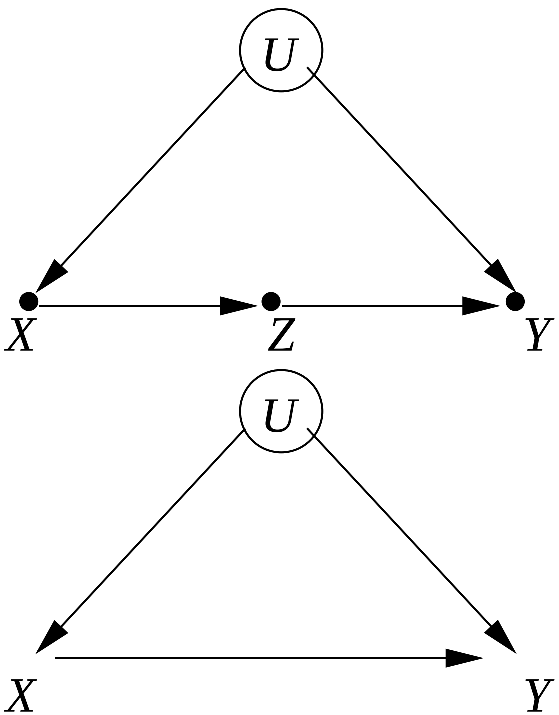
Identifiability Problem

V: observables, U: unobservables.

- Input: causal graph $+ P(v)$.
- Can $P_t(s)$ be computed uniquely from $P(v)$?

Semi-Markovian Models

- Each hidden variable is a root node and has exactly two observed children.



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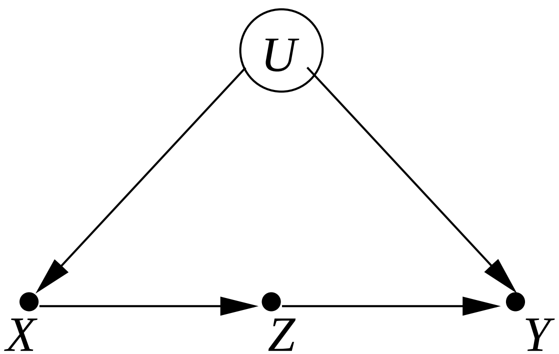

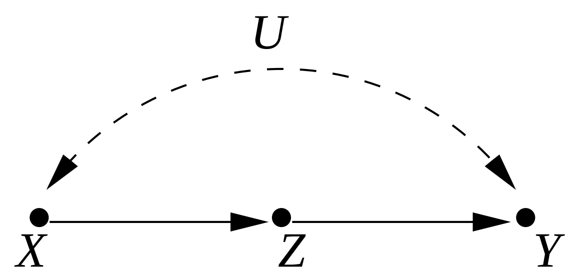

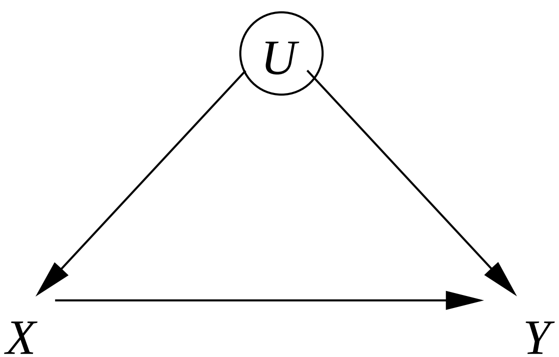

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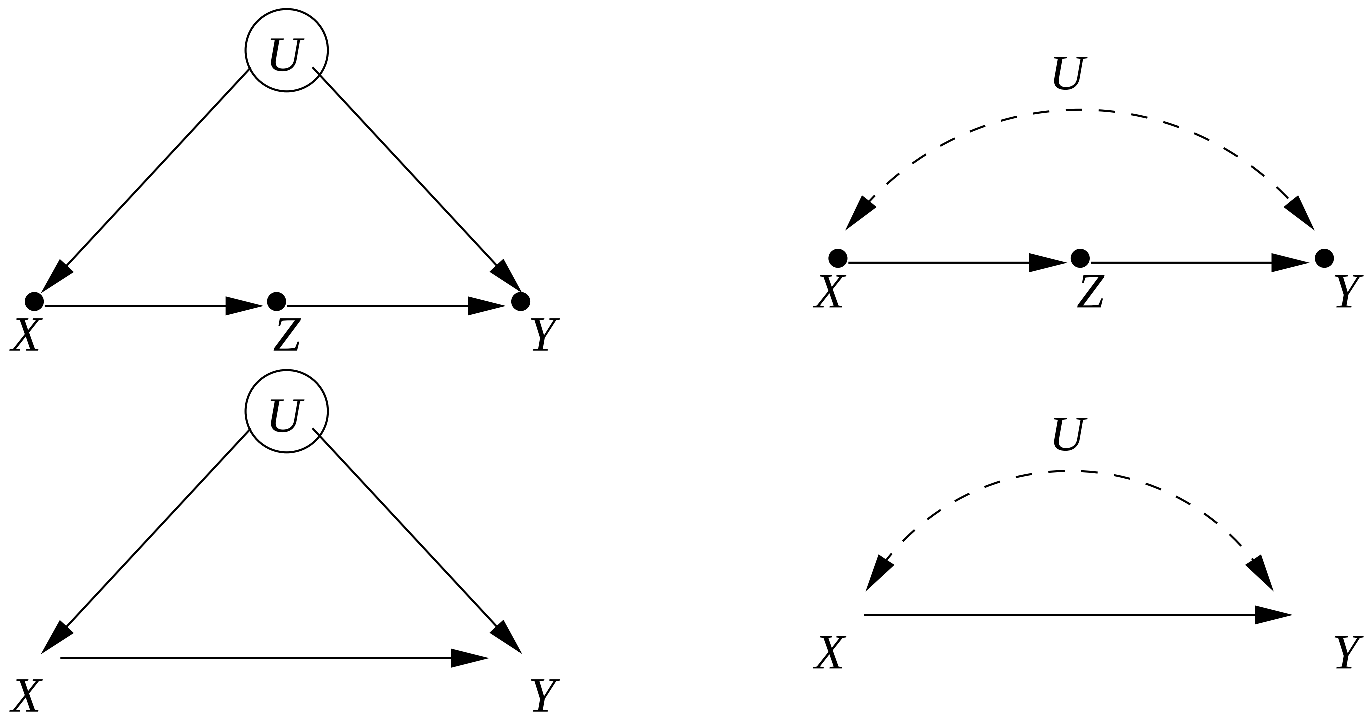

Previous Work

- *do*-calculus (Pearl, 1995)

Rule 1: Ignoring observations

$$P_x(y|z,w) = P_x(y|w) \quad \text{if } (Y \perp\!\!\!\perp Z | X, W)_{G_{\overline{X}}}$$

Rule 2: Action/observation exchange

$$P_{x,z}(y|w) = P_x(y|z,w) \quad \text{if } (Y \perp\!\!\!\perp Z | X, W)_{G_{\overline{X}\underline{Z}}}$$

Rule 3: Ignoring actions

$$P_{x,z}(y|w) = P_x(y|w) \quad \text{if } (Y \perp\!\!\!\perp Z | X, W)_{G_{\overline{X}\,\overline{Z(W)}}}$$

Previous Work – Graphical Criteria

- $P_x(s)$:
  - Back-door criterion (Pearl, 1993).
  - Front-door criterion (Pearl, 1995).
  - Galles and Pearl (1995) criterion.
- $P_t(s)$:
  - Pearl and Robins (1995).
  - Robins (1997).
  - Kuroki and Miyakawa (1999).
- Summarized in (Pearl 2000, chapters 3 and 4).

My Work

- A new method of identifying causal effects.
- Simpler and more powerful graphical criteria.
- Procedures for deriving $P_t(s)$.
- Beyond semi-Markovian models.

The Criterion for Identifying $P_x(v)$

Theorem $P_x(v)$ *is identifiable if and only if there is no bidirected path connecting* X *to any of its children.*

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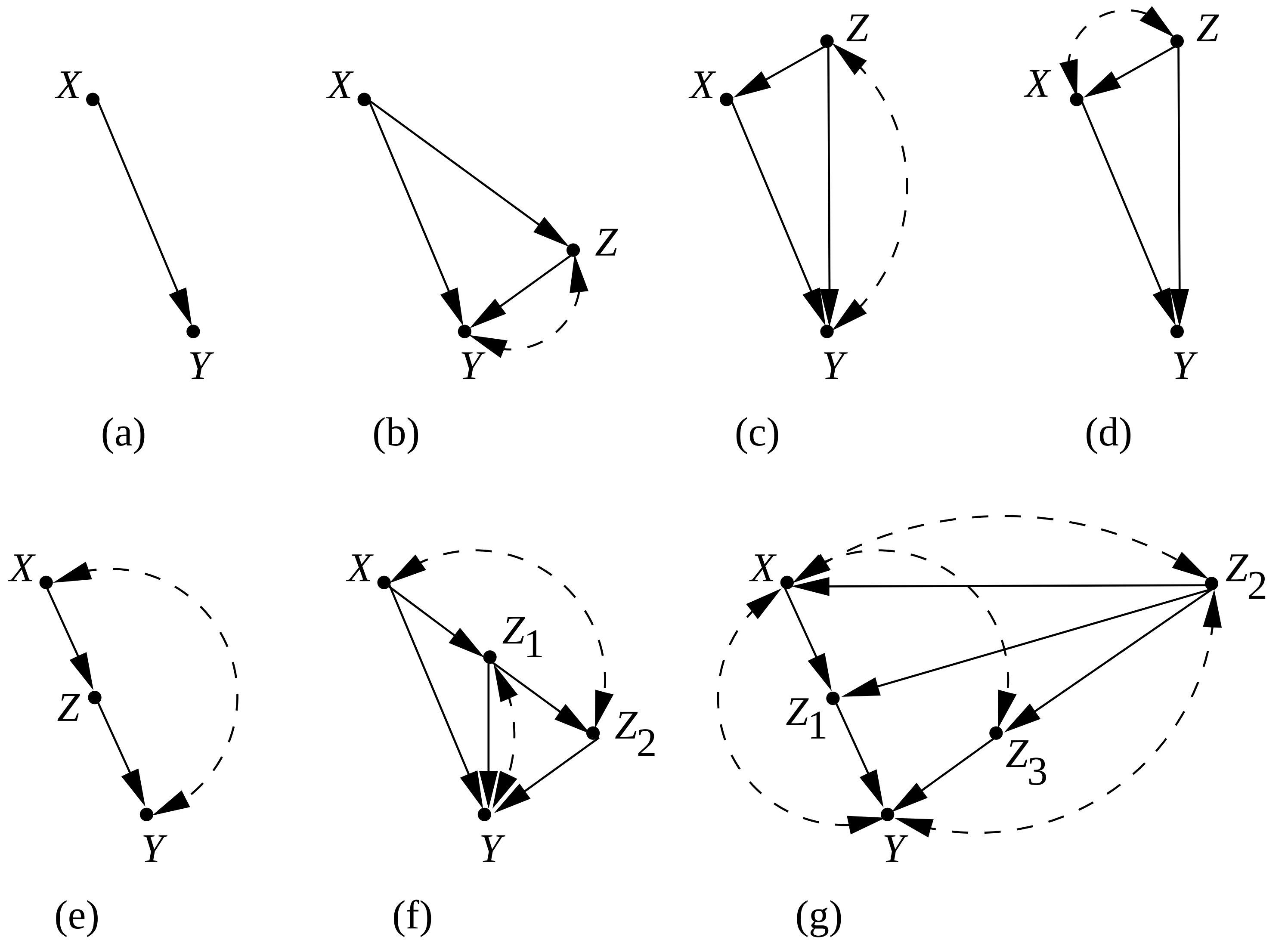
When the condition is satisfied,

- A closed-form expression for $P_x(v)$ is available.

Examples (Pearl 2000, Figure 3.8)

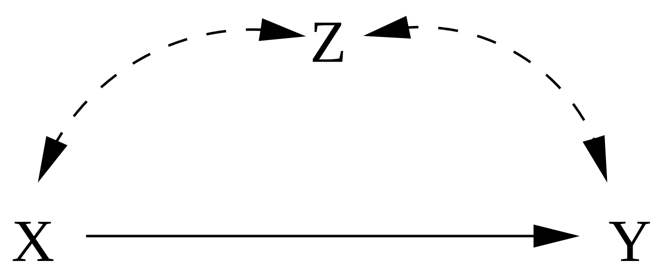


The Identification of $P_x(s)$

- Whenever $P_x(v)$ is identifiable, so is $P_x(s)$.

However,

- There are obvious cases where $P_x(v)$ is not identified and still $P_x(s)$ is identified.



A Criterion for Identifying $P_x(s)$

Theorem $P_x(s)$ *is identifiable if there is no bidirected path connecting* X *to any of its children in* $G_{An(S)}$.

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Theorem $P_x(s)$ *is identifiable if there is no bidirected path connecting* X *to any of its children in* $G_{An(S)}$.

- This simple criterion covers all existing criteria in the literature: back-door, front-door, and (Galles and Pearl 1995) criteria.

Computing $P_t(s)$

- A procedure for computing $P_t(s)$ is developed.

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The procedure

- Finds an expression for $P_t(s)$, or
- Reduces the problem to that in a type of graphs that might be unidentifiable (open problem).

Open Problem

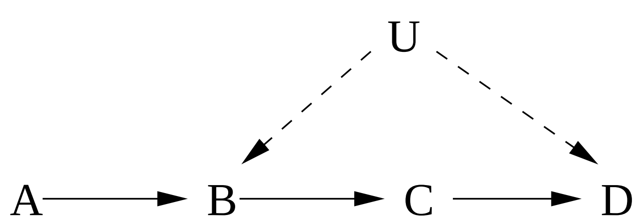
- Is $P_{v \setminus s}(s)$ identifiable? If
  1. All variables in G_S are connected by bidirected paths; and
  2. All variables in G are connected by bidirected paths; and
  3. All variables in $V \setminus S$ are ancestors of S.

Summary

- We developed a new method for inferring causal effects.
- We show some powerful graphical criteria for identifying causal effects.
- We developed a procedure that systematically identifies causal effects.

Other Work

On the Testable Implications of Causal Models with Hidden Variables, Tian, J. and Pearl, J., UAI'02.



- $P(a,b,c,d)$ must satisfy

$$\sum_b P(d|a,b,c)P(b|a) = f(c,d)$$

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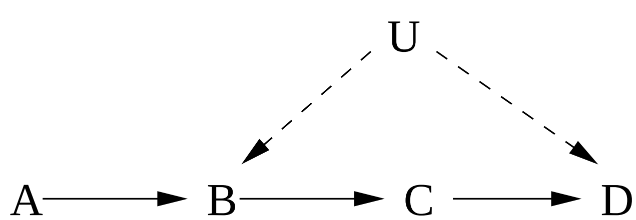

- $P(a,b,c,d)$ must satisfy

$$\sum_b P(d|a,b,c)P(b|a) = f(c,d)$$

- We developed a procedure that systematically identifies functional constraints.

Other Work

A new characterization of the experimental implications of causal Bayesian networks, Tian, J. and Pearl, J., AAAI'02.

- Given a collection of interventional distributions $P_t(v)$, is the collection compatible with some underlying causal BN?

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- Given a collection of interventional distributions $P_t(v)$, is the collection compatible with some underlying causal BN?
- A Complete Axiomatization:
  1. Effectiveness $P_t(t) = 1$
  2. Markov $P_{v \setminus (s_1 \cup s_2)}(s_1, s_2) = P_{v \setminus s_1}(s_1) P_{v \setminus s_2}(s_2)$
  3. Recursiveness
     $(X_0 \rightsquigarrow X_1) \wedge \ldots \wedge (X_{k-1} \rightsquigarrow X_k) \Rightarrow \neg(X_k \rightsquigarrow X_0)$

Conclusion

- We developed a procedure that systematically identifies causal effects.
- We developed a procedure that systematically identifies functional constraints induced by causal Bayesian networks with hidden variables.
- We offer a complete characterization of interventional distributions induced by causal Bayesian networks.