
CS264A: Automated Reasoning

Problem Set #2, Fall 06

1. Show the termination tree for DPLL when run on the following KB, assuming that variables are tested according to the order A, B, C, D, E and true expanded before false:

$$\Delta = \begin{array}{l} 1. A \wedge D \Rightarrow E \\ 2. C \Rightarrow D \\ 3. D \Rightarrow \neg E \\ 4. B \wedge \neg C \Rightarrow D \end{array}$$

Note that DPLL does not use conflict-directed backtracking.

2. Show a trace of DPLL+ on the above KB, assuming that decisions are made according to the constraints given above. At each conflict, show the decision sequence, implication graph, conflict-drive clause, and its assertion level. Perform two traces of DPLL+: one assuming that conflict-driven clauses are generated using the first UIP method of Section 3.6.2, and a second trace assuming the method given in Section 3.5.
3. Consider the following knowledge base.

$$\Delta = (A \vee B) \Rightarrow C, \neg D \Rightarrow \neg C, D \Rightarrow E.$$

Show how you can count the number of models of Δ using CDPLL and draw the termination tree. Assume that you are expanding variables according to the order A, B, C, D, E and always expand true before false.

4. Show that directed resolution can be applied to any KB containing only binary clauses in $O(n^3)$ time, where n is the number of variables.
5. Consider the following knowledge base:

$$\Delta = P_1 \vee P_2 \vee P_3, P_1 \Rightarrow Q, P_2 \Rightarrow Q, P_3 \Rightarrow Q.$$

- a. Convert Δ into clausal form.
- b. Apply directed resolution to the clausal form using the order P_1, P_2, P_3, Q .
- c. Construct a decision tree for Δ and use it to count the number of models of Δ .
- d. Consider a generalization of Δ :

$$\Delta_n = P_1 \vee \dots \vee P_n, P_1 \Rightarrow Q, P_2 \Rightarrow Q, \dots, P_n \Rightarrow Q.$$

How many models does Δ_n have? Why?

- e. What is the treewidth of the connectivity graph of Δ_n 's clausal form?