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## CS264A: Automated Reasoning

Problem Set #1, Fall 06

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1. Show that the following sentences are consistent by identifying a world which satisfies each sentence:

- $(A \Rightarrow B) \wedge (A \Rightarrow \neg B)$ .
- $(A \vee B) \Rightarrow (\neg A \wedge \neg B)$ .

2. Show that the following sentences are valid by showing that each is true at every world:

- $(A \wedge (A \Rightarrow B)) \Rightarrow B$ .
- $(A \Rightarrow B) \Rightarrow (\neg B \Rightarrow \neg A)$ .

3. Prove that the sentence  $\alpha \Leftrightarrow \beta$  is valid iff  $\alpha$  is equivalent to  $\beta$ . Hint: write down the formal definition of each statement first and then prove equivalence.

4. Prove from the definitions of Boolean quantifiers  $\exists$  and  $\forall$  that (a)  $\exists P(\Delta \vee \Gamma)$  is equivalent to  $(\exists P\Delta) \vee (\exists P\Gamma)$ , and (b)  $\forall P(\Delta \wedge \Gamma)$  is equivalent to  $(\forall P\Delta) \wedge (\forall P\Gamma)$ .

5. Convert the following knowledge base to clausal form:

$$\Delta = \{A \wedge B \Rightarrow \neg C, A \vee B \Rightarrow D, \neg(C \vee D) \Rightarrow E\}.$$

6. What is the characteristic Boolean function of the following knowledge base:

$$\Delta = \{A \Rightarrow (B \vee C)\}.$$

7. Show that if a quantified propositional sentence is closed (that is, all variables are quantified over), then the sentence is either inconsistent or valid.

8. Show that if we have a polynomial procedure for model counting, and another for clausal entailment on a knowledge base  $\Gamma$ , then we have a polynomial procedure for testing the equivalence between  $\Gamma$  and any CNF  $\Delta$ .

9. Show using resolution that  $D \vee E$  is entailed by the knowledge base:

$$\{A \Rightarrow B, A \Rightarrow \neg C, \neg D \Rightarrow \neg B \wedge C, \neg A \Rightarrow E\}.$$